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THREE DIMENSIONAL TRANSVERSE STRENGTH FOR SUPER TANKERS

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THREE DIMENSIONAL TRANSVERSE STRENGTH FOR SUPER TANKERS

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under the direction of
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Presented as a partial requirement for the degree of Master of Engineering in Naval Architecture in the University of California, Berkeley.

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I. INTRODUCTION

Today, naval architects are greatly interested in the design of super tankers. However, many difficulties arise in design when the strength of the super tanker is considered from an economic point of view. The transverse strength problem is apparently one of these difficulties.

There are several ways in analysing the transverse strength problem. In each method, the two-dimensional approach is simpler, but the three-dimensional approach is so complicated that hardly no one wishes to use it. Therefore, some approximate is devised and sufficient results are achieved. But still some architects wish to know the three-dimensional effect on transverse strength analysis.

Some prominent naval architects suggested solving the three-dimensional strength calculation but most of them did not apply numerical computations to existing complicated ships, for even though the computor is available, it is still no easy task. So many variables exist it is very tedious work to put them into computors.

In this paper we are going to consider these tedious problems by using FORTRAN 7090.

As previously mentioned, there are many ways to solve the three-dimensional transverse strength problem and each one has its own merits, although they will give almost the same level of accuracy.

Since some approaches are so complicated and give the same level of accuracy as more simple approaches, they become useless. Therefore,

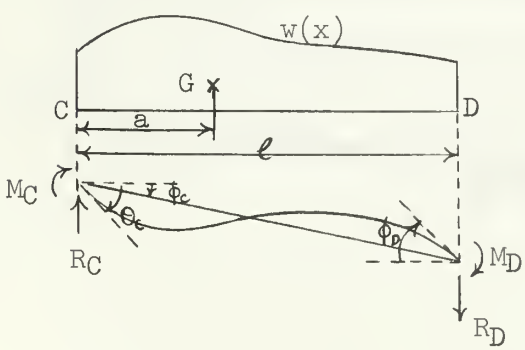
we are going to deal with a well-known and more simple approach--the slope-deflection method--to attack these three-dimensional calculations and examine its effect.

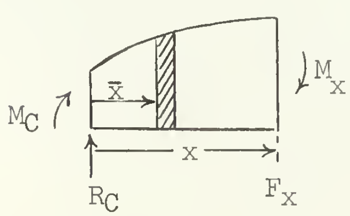
In large tankers especially, each span has such deep depth-breadth ratios that we suppose shear force contributes more than usual, therefore, we take into account shear force effect in slope deflection equation.

Eventually, we will check our computed results by means of slope-deflection method with one of the fixed point methods which was given in our class by Professor Schade. This fixed point method differs in assumption from the slope-deflection method, but we can make this fixed point assumption comparable with our slope-deflection assumption by restricting our loading conditions. Although we are dealing with the fixed point method, it will not give us shear force effect and three-dimensional effect. We can see, therefore, only the differences between those two: the slope-deflection method with shear force effect and the fixed method without. They will lead us to some interesting conclusions.

II. BASIC SLOPE-DEFLECTION EQUATION CONSIDERED WITH SHEAR EFFECT

(1) Simple beam





$$M_x = -M_C\left(1 - \frac{x}{\ell}\right) + M_D\frac{x}{\ell} - \frac{W(\ell - a)}{\ell}x + x\int_0^x w(x)dx - \int_0^x w(x)xdx$$

Where $\int_0^x w(x)(x - \bar{x})d\bar{x}$

$$= x\int_0^x w(x)dx - \int_0^x w(x)xdx$$

$$F_x = -\frac{1}{\ell}(M_C + M_D) + \frac{W(\ell - a)}{\ell} - \int_0^x w(x)dx$$

Here $W = \int_0^\ell w(x)dx$

$$a = \frac{1}{W}\int_0^\ell w(x)xdx$$

Energy equation of the beam

$$U = \frac{1}{2EI_o}\int_o^{\ell} M^2{}_x dx + \frac{1}{2GA}\int_o^{\ell} F^2{}_x dx$$

Applying Castigliano theorem

$$\frac{\partial U}{\partial M_C} = \theta_C - \phi_o \qquad \frac{\partial U}{\partial M_D} = \theta_D - \phi_o$$

put

$$\eta = 6EI_o/AG\ell^2$$

Then

$$M_C = \frac{2EI_o}{(2\eta+1)\ell}\left\{(\eta+2)\theta_C + (1-\eta)\theta_D - 3\phi_o\right\} + \frac{1}{2\eta+1}\left\{-(\eta+2)C_1 + C_2\right\}$$

$$M_D = \frac{2EI_o}{(2\eta+1)\ell}\left\{(1-\eta)\theta_C + (\eta+2)\theta_D - 3\phi_o\right\} + \frac{1}{2\eta+1}\left\{(\eta-1)C_1 + C_2\right\}$$

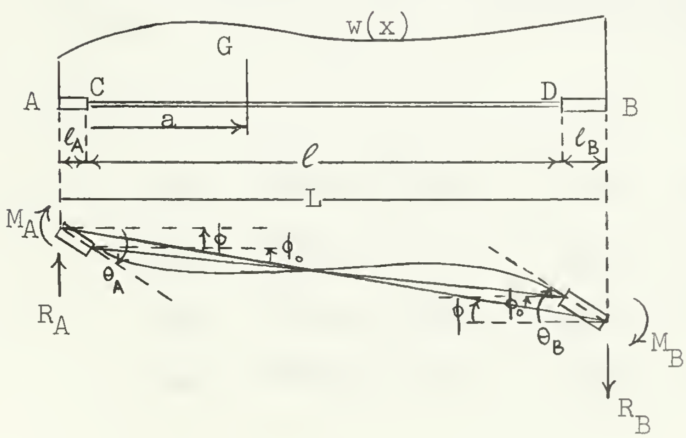
$$R_C = -\frac{6EI_o}{(2\eta+1)\ell^2}(\theta_C + \theta_D - 2\phi_o) + \frac{3C_1-2C_2}{(2\eta+1)\ell} + \frac{W}{\ell}(\ell-a)$$

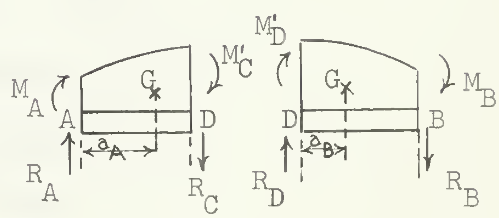
$$R_D = -\frac{6EI_o}{(2\eta+1)\ell^2}(\theta_C + \theta_D - 2\phi_o) + \frac{3C_1-2C_2}{(2\eta+1)\ell} - \frac{aW}{\ell}$$

Where

$$\begin{cases} C_1 = Wa - \frac{1}{\ell}\int_o^{\ell} w(x)x^2 dx \\ C_2 = Wa - \frac{1}{\ell^2}\int_o^{\ell} w(x)x^3 dx \end{cases}$$

(2) Simple beam with bracket at both ends





$M_C' = -M_C$ and $M_D' = -M_D$

$$R_A = R_C + W_A$$

$$R_B = R_D - W_B$$

$$M_A = M_C - R_C \ell_A - W_A a_A$$

$$M_B = M_D - R_D \ell_B + W_B(\ell_B - a_B)$$

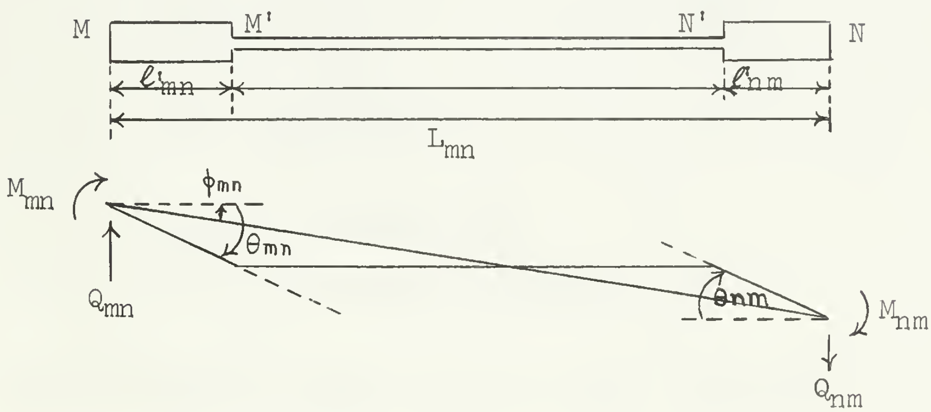
Where

$$\begin{cases} W_i = \int_0^{\ell_i} w(x_i)dx_i \\ a_i = \dfrac{\int_0^{\ell_i} w(x_i)x_i dx_i}{W_i} \end{cases} \qquad i = A, B$$

also

$$\begin{cases} \Theta_C = \Theta_A \qquad \Theta_D = \Theta_B \\ \phi_o = \dfrac{1}{\ell}(L\phi - \ell_A\Theta_A - \ell_B\Theta_B) \end{cases}$$

Using all of these results and the result of (1), we can easily get the following basic formula using the following notation.



$$M_{mn} = 2EK_o k_{mn}(2\alpha_{mn}\Theta_{mn} + \beta_{mn}\Theta_{nm} - 3\delta_{mn}\phi_{mn}) + C_{mn}$$

$$M_{nm} = 2EK_o k_{mn}(\beta_{mn}\Theta_{mn} + 2\alpha_{nm}\Theta_{nm} - 3\delta_{nm}\phi_{mn}) + C_{nm}$$

$$Q_{mn} = -6EK_o\gamma_{mn}(\mu_{mn}\Theta_{mn} + \mu_{nm}\Theta_{nm} - 2\nu_{mn}\phi_{mn}) + D_{mn}$$

$$Q_{nm} = -6EK_o\gamma_{nm}(\mu_{mn}\Theta_{mn} + \mu_{nm}\Theta_{nm} - 2\nu_{mn}\phi_{mn}) + D_{nm}$$

Where

$$\alpha_{mn} = \frac{1 + \frac{\eta_{mn}}{2} + 3a_{mn} + 3a^2_{mn}}{2\eta_{mn} + 1}$$

$$\alpha_{mn} = \frac{1 + \frac{\eta_{mn}}{2} + 3b_{mn} + 3b^2_{mn}}{2\eta_{mn} + 1}$$

$$\beta_{mn} = \frac{1 - \eta_{mn} + 3(a_{mn} + b_{mn}) + 6a_{mn}b_{mn}}{2\eta_{mn} + 1}$$

$$\delta_{mn} = \mu_{mn}d_{mn} \qquad \delta_{nm} = \mu_{nm}d_{mn}$$

$$\mu_{mn} = \frac{1 + 2a_{mn}}{2\eta_{mn} + 1} \qquad \mu_{nm} = \frac{1 + 2b_{mn}}{2\eta_{mn} + 1}$$

$$a_{mn} = \ell'_{mn}/\ell_{mn} \qquad b_{mn} = \ell'_{mn}/\ell_{mn}$$

$$\delta_{mn} = \frac{d_{mn}}{2\eta_{mn} + 1} \qquad d_{mn} = L_{mn}/\ell_{mn}$$

$$\gamma_{mn} = k_{mn}/\ell_{mn} \qquad k_{mn} = \frac{I_{mn}/\ell_{mn}}{K_o}$$

$$\eta_{mn} = \frac{6EI_{mn}}{Aw_{mn}G\ell^2_{mn}}$$

$Aw_{mn} \longrightarrow$ Area of web

$I_{mn} \longrightarrow$ Moment of inertia

and the constant terms C_{mn} , C_{nm} , D_{mn} , D_{nm} are:

$$C_{mn} = \frac{1}{2\eta_{mn} + 1}\left\{(1 + 2a_{mn})C_2 - (\eta_{mn} + 2 + 3a_{mn})C_1\right\}$$
$$- a_{mn}(\ell_{mn} - g_{mn})W_{mn} - g'_{mn}W'_{mn}$$

$$C_{nm} = \frac{1}{2\eta_{mn} + 1}\left\{(1 + 2b_{mn})C_2 - (1 - \eta_{mn} + 3b_{mn})C_1\right\}$$
$$- b_{mn}g_{mn}W_{mn} + (\ell'_{nm} - g'_{nm})\,W'_{nm}$$

$$D_{mn} = \frac{3C_1 - 2C_2}{(2\eta_{mn} + 1)\ell_{mn}} + \frac{W_{mn}}{\ell_{mn}} (\ell_{mn} - g_{mn}) + W'_{mn}$$

$$D_{nm} = \frac{3C_1 - 2C_2}{(2\eta_{mn} + 1)\ell_{mn}} \quad \frac{g_{mn}}{\ell_{mn}} W_{mn} - W'_{nm}$$

Here

$$C_1 = W_{mn}g_{mn} - \frac{1}{\ell_{mn}} \int_o^{\ell_{mn}} w(x)x^2dx$$

$$C_2 = W_{mn}g_{mn} - \frac{1}{\ell^2_{mn}} \int_o^{\ell_{mn}} w(x)x^3dx$$

$W_{mn} \longrightarrow$ sum of the load on M'N'

$W'_{mn} \longrightarrow$ sum of the load on MM'

$W'_{nm} \longrightarrow$ sum of the load on N'N

g_{mn} , g'_{mn} , g'_{nm} is the distance from left end edge to the center of load for each span.

For the purpose of our calculations, we only need two kinds of loading spans. They are shown on Table I and Table II.

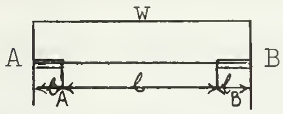
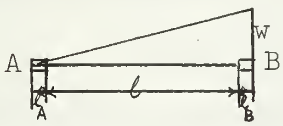
	C_A	C_B
	$-\frac{w\ell^2}{12}(1 + 6a + 6a^2)$	$\frac{w\ell^2}{12}(1 + 6b + 6b^2)$
	$-\frac{w\ell^2}{d}\left[\frac{a^2}{6}(2a + 3) + \frac{1}{1+2\eta}\left(\frac{1+7a}{30} + \frac{1+6a}{12}\eta\right)\right]$	$\frac{w\ell^2}{d}\left[\frac{b^2}{6}(3d-2b) + \frac{a}{12}(1+6b) + \frac{1}{1+2\eta}\left(\frac{1+7b}{20} + \frac{1+8b}{12}\eta\right)\right]$

Table 1

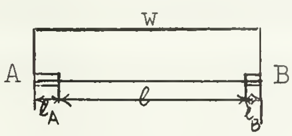
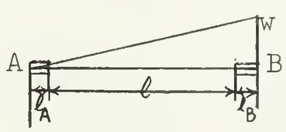
	D_A	D_B
	$\frac{w\ell}{2}(1 + 2a)$	$-\frac{w\ell}{2}(1 + 2b)$
	$\frac{w\ell}{d}\left[\frac{a}{2}(a + 1) + \frac{1}{1+2\eta}\left(\frac{3}{20} + \frac{\eta}{3}\right)\right]$	$-\frac{w\ell}{d}\left[db - \frac{b^2}{2} + \frac{a}{2} + \frac{1}{1+2\eta}\left(\frac{7}{20} + \frac{2\eta}{3}\right)\right]$

Table 2

III. SETTING UP SIMULTANEOUS EQUATION

A. Assumption

Once the designer has made the decision to consider the transverse frame of a ship, he is then faced with the task of deciding on simplifying assumptions. The main objective of these assumptions is to create a fictitious structure and condition of loading, which will reduce the amount of labor in performing the structural analysis without compromising too severely the validity of the numerical answers.

(1) One of the factors a designer must consider is the degree of dimensional similitude he can retain in the structure he selects for his computational work. For example, he must decide how much of the shell plating should be included in the analysis of a transverse frame as effective material. We are going to use Effective Breadth Conception in Ship Structure Design, published in 1954 by Professor Schade.

There are several other assumptions to be applied in this paper.

(2) The relative angle initially formed by the tangents to the elastic curve at a point where two or more members are joined are assumed to remain unchanged after loading; that is, if two members are originally joined at right angles, the angle formed by the tangents to the elastic curve after the loads are applied will remain 90°.

(3) Joints of 1-1 (or 9-2) and 4-2 (or 13-2) are fixed in space but free to rotate in three-dimensional analysis and additionally 9-1 (or 14-2), 15-2 (or 13-1), 14-1 and 15-1 are fixed in space in two-dimensional analysis (see Fig. 2-2).

B. Simultaneous Equation

Before setting up simultaneous equations we must investigate the number of unknown variables. From the basic slope equation we can take

up the following variables as unknown variables.

1. $\theta1(1) = \theta2(9) = X(1)$
2. $\theta1(9) = \theta1(5) = \theta2(14) = X(2)$
3. $\theta2(1) = \theta2(10) = \theta1(2) = X(3)$
4. $\theta2(5) = \theta1(10) = \theta1(6) = X(4)$
5. $\theta2(2) = \theta2(11) = \theta1(3) = X(5)$
6. $\theta2(6) = \theta1(11) = \theta1(7) = X(6)$
7. $\theta2(3) = \theta2(12) = \theta1(4) = X(7)$
8. $\theta2(7) = \theta1(12) = \theta1(8) = X(8)$
9. $\theta2(4) = \theta2(13) = X(9)$
10. $\theta2(8) = \theta1(13) = \theta2(15) = X(10)$
11. $\phi(1) = \phi(5) = X(11)$
12. $\phi(2) = \phi(6) = X(12)$
13. $\phi(3) = \phi(7) = X(13)$
14. $\phi(4) = \phi(8) = X(14)$
15. $\phi(9) = \phi(10) = \phi(11) = \phi(12) = \phi(13) = X(15)$
16. $\phi(14) = X(16)$
17. $\phi(15) = X(17)$

Now we have 17 unknown variables in one transverse frame. Therefore we have to have at least 17 simultaneous equations for each frame.

1. $M1(1) + M2(9) = 0$
2. $M1(9) + M1(5) + M2(14) = 0$
3. $M2(1) + M2(10) + M1(2) = 0$
4. $M2(5) + M1(10) + M1(6) = 0$
5. $M2(2) + M2(11) + M1(3) = 0$
6. $M2(6) + M1(11) + M1(7) = 0$

7. $M2(3) + M2(12) + M1(4) = 0$

8. $M2(7) + M1(12) + M1(8) = 0$

9. $M2(4) + M2(13) = 0$

10. $M2(8) + M1(13) + M2(15) = 0$

11. $Q1(6) + Q1(2) - Q2(1) - Q2(5) = 0$

12. $Q1(7) + Q1(3) - Q2(2) - Q2(6) = 0$

13. $Q1(8) + Q1(4) - Q2(7) - Q2(3) = 0$

14. $X(11) \times SPAN(1) + X(12) \times SPAN(2) + X(13) \times SPAN(3) + X(14) \times SPAN(4) = 0$

For the three dimensional analysis we can set up the following three more equations.

15. $Y_B = X(15) \times SPAN(13) + X(17) \times SPAN(15)$

16. $Y_D = X(15) \times SPAN(9) + X(16) \times SPAN(14)$

17. $Y_L = X(15) \times SPAN(13)$

Here Y_B , Y_D and Y_L are deflection of bottom and deck longitudinals and longitudinal BHD. We suppose that longitudinals are only acted by the concentrated load at joints with transverse frames. Then acting load P_B , P_D and P_L are expressed as follows:

$$P_B = 2Q1(15)$$

$$P_D = 2Q1(14)$$

$$P_L = Q1(9) + Q1(10) + Q1(11) + Q1(12) + Q1(13) - Q2(15) - Q2(14)$$

For the two dimensional analysis we need only to solve 14 simultaneous equations with unknown variables 14 by setting simply X(15), X(16) and X(17) to zero in the above equations.

In this paper, for the practical numerical computation, we will take an existing Japanese built Tanker as an example. She has 47,000 tons Dead Weight and one of the tank's length is 503.936" and height 604.329" and its breadth is 1188.974". One of the web frame's

scantling is shown in Fig. 1 and three web frames placed in a length of tank.

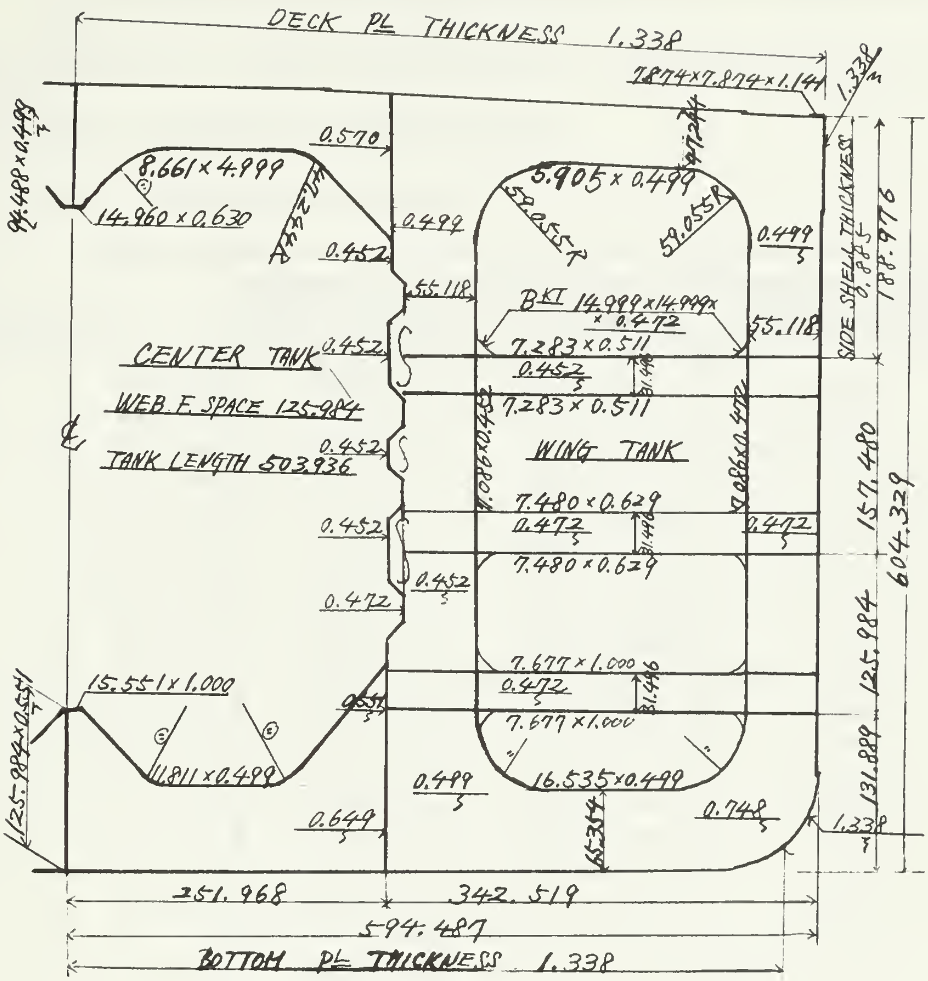


FIG. 1 TRANS. WEB FRAME

IV. PRELIMINARY CALCULATION

A. Section modulus calculation

From the effective breadth conception curve CASE (III), we can easily find out ρ : Effectiveness ratio. From these data we can calculate moment of inertia and neutral axis.

The results are as follows:

SPAN No.	ρ	MOMENT OF INERTIA OF THE SECTION	RATIO OF INERTIA	DISTANCE FROM N.A. TO FREE FLANGE
1	0.570	30828.851	0.913	47.240
2	0.350	25839.827	0.794	42.339
3	0.350	25839.827	0.794	42.339
4	0.440	45197.227	0.822	41.335
5	0.635	23604.205	0.891	42.189
6	0.410	20716.733	0.780	38.867
7	0.410	21014.376	0.784	39.184
8	0.485	60847.290	0.814	33.413
9	0.870	22181.710	0.985	44.287
10	1.000	3083.156	1.000	16.259
11	1.000	3656.705	1.000	16.377
12	1.000	5282.347	1.000	16.748
13	0.870	84741.346	0.974	55.841
14	0.735	24303.928	0.963	43.268
15	0.735	60303.312	0.945	57.421

Table 3

In inches

SPAN No.	SPAN LENGTH L	ℓ	$\frac{\ell_1}{\ell_2}$
1	198.454	122.859	$\frac{58.367}{17.228}$
2	125.984	90.967	$\frac{17.372}{17.645}$
3	125.984	89.928	$\frac{17.645}{18.411}$
4	137.987	40.587	$\frac{17.843}{79.554}$
5	198.454	120.428	$\frac{60.574}{17.452}$
6	125.984	90.536	$\frac{17.596}{17.892}$
7	125.984	89.456	$\frac{17.857}{18.671}$
8	137.984	39.287	$\frac{17.955}{80.745}$
9	301.006	176.205	$\frac{61.037}{63.764}$
10	301.006	196.093	$\frac{52.051}{52.862}$
11	301.006	196.840	$\frac{51.670}{52.496}$
12	301.006	198.488	$\frac{50.850}{51.668}$
13	301.006	190.620	$\frac{54.579}{55.807}$
14	280.539	233.377	$\frac{11.062}{36.100}$
15	280.539	236.469	$\frac{12.451}{31.619}$

Table 4

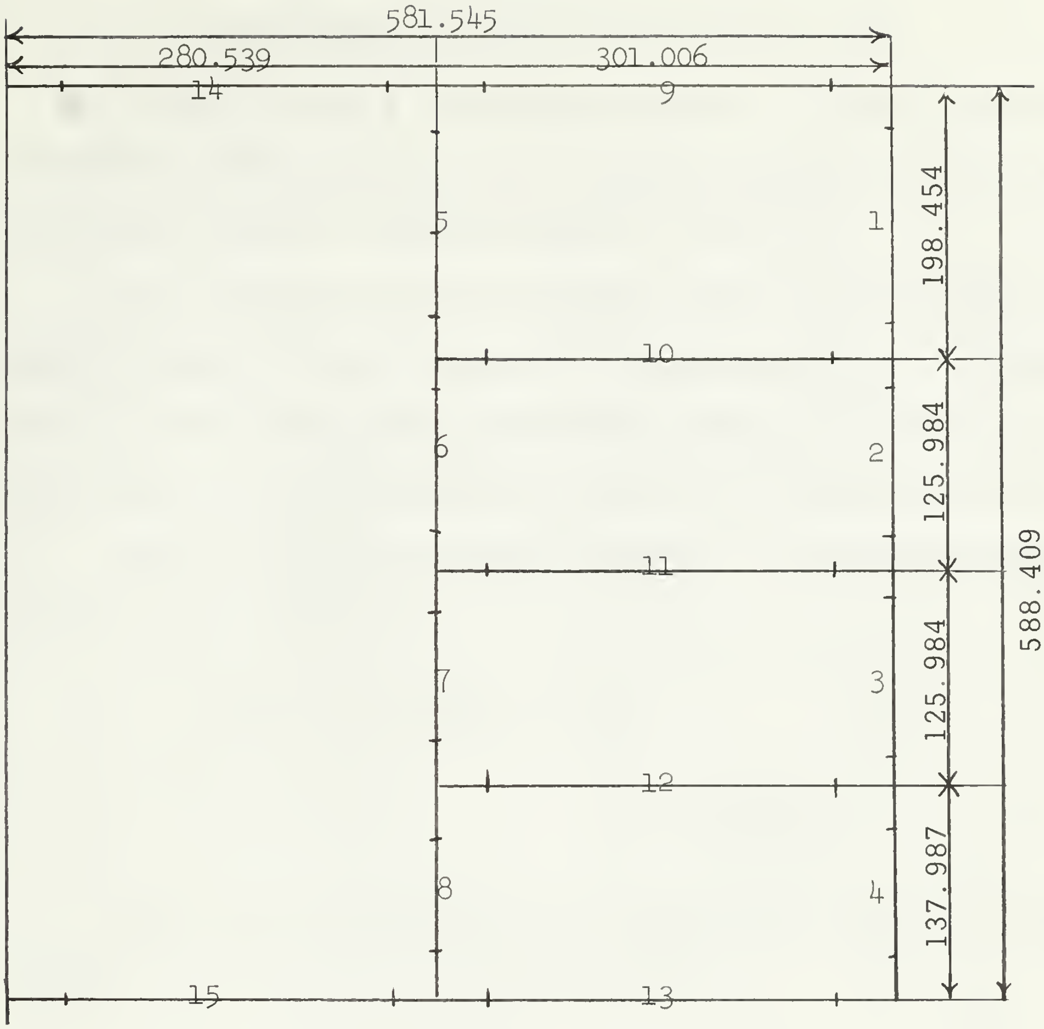


Fig. 2-1 Designation of span number

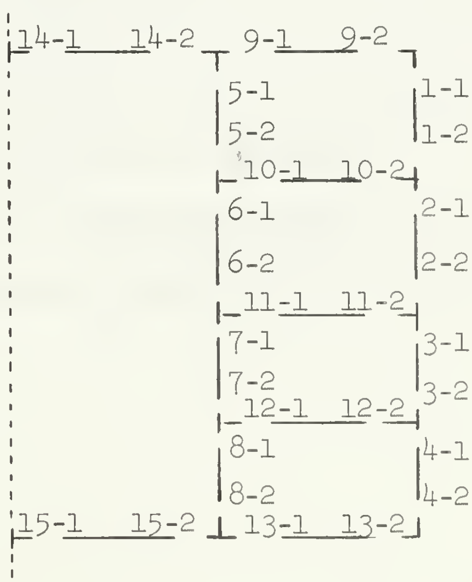


Fig. 2-2 End point designation

V. SOLVING A SIMULTANEOUS EQUATION USING COMPUTOR 7090

Before solving a simultaneous equation we decide to solve two different loading conditions: (1) favorable loading condition (see Fig. 3), (2) most critical loading condition (see Fig. 4).

For the girder deflection we must also think about shear deformation because of having relatively short span length and deep depth. For this purpose we can find deflection equations of simple beam at the condition of both ends fixed and concentrated at some span point.

One can use the next simple equation for the concentrated load at the center of the beam.

$$W_p = - \frac{1}{192} \frac{b^3 P}{E I_b} (1 + C_p)$$

$$C_p = \frac{48 E I_b}{b^2 G A_b}$$

When the concentrated load is acting at the point of 1/4 span length, deflection can be expressed as follows:

$$W_p = - \frac{1}{512} \frac{9}{8} \frac{b^3 P}{E I_b} (1 + 2C_p)$$

Where W_p...Deflection at the point of load applied

A_b...Web area of b beam

P ...Concentrated load

For the computed deflection, see curves 4-1, 4-2 and 4-3.

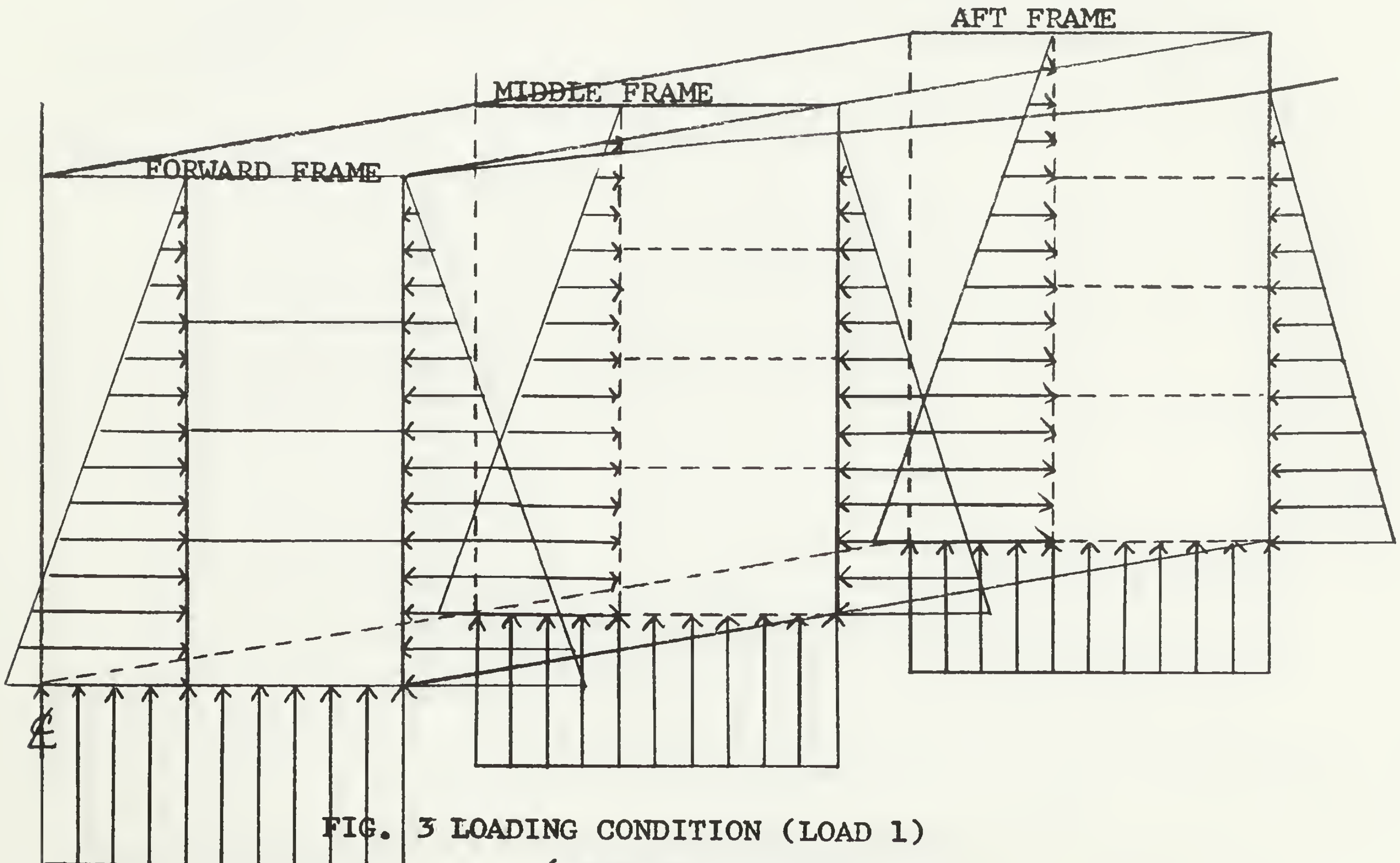


FIG. 3 LOADING CONDITION (LOAD 1)
(BALLASTED CENTER TANK)

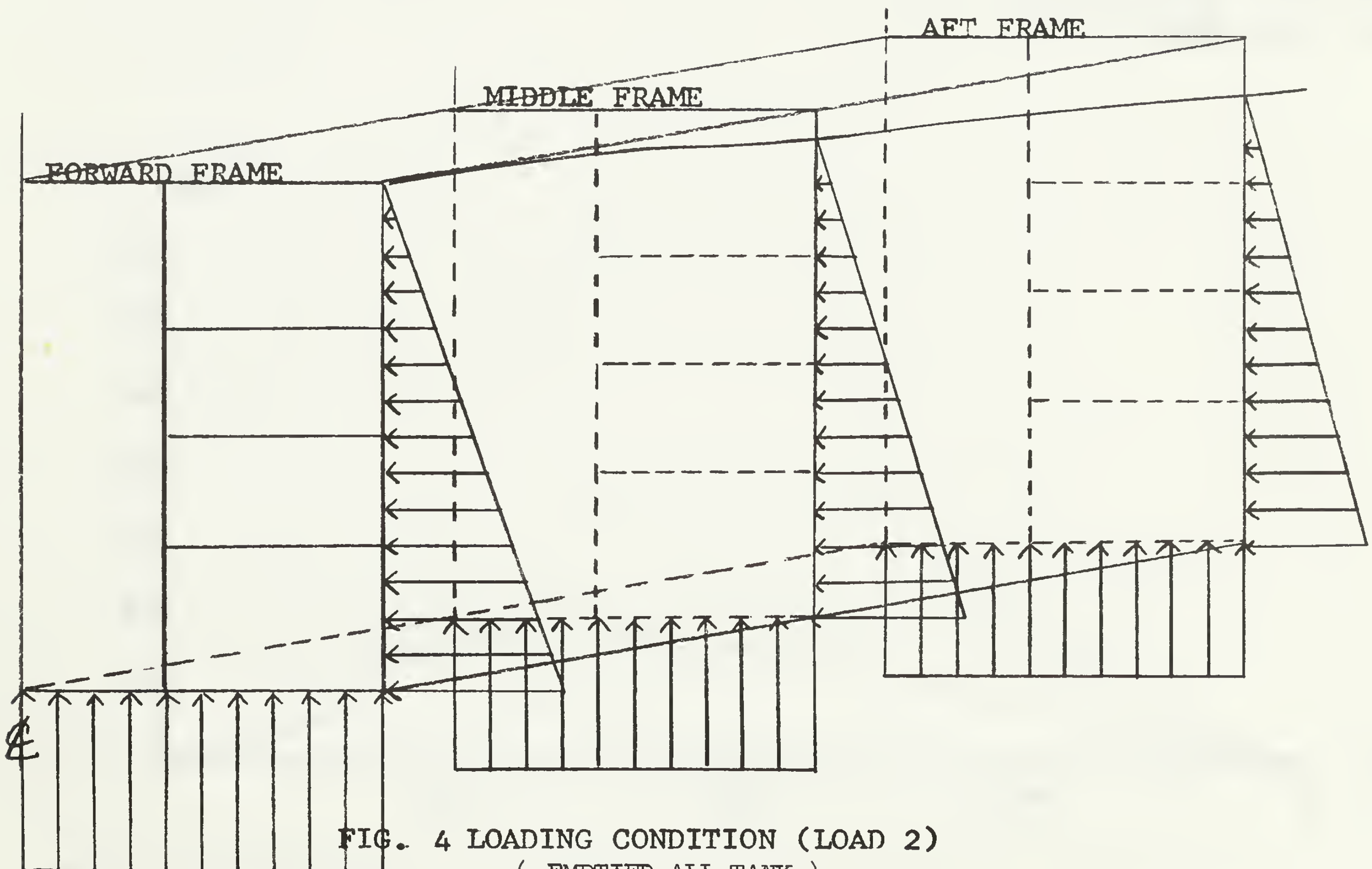


FIG. 4 LOADING CONDITION (LOAD 2)
(EMPTIED ALL TANK)

FIG.4-1 UNIT LOAD DEFLECTION OF LONGITUDINAL CENTER GIRDER

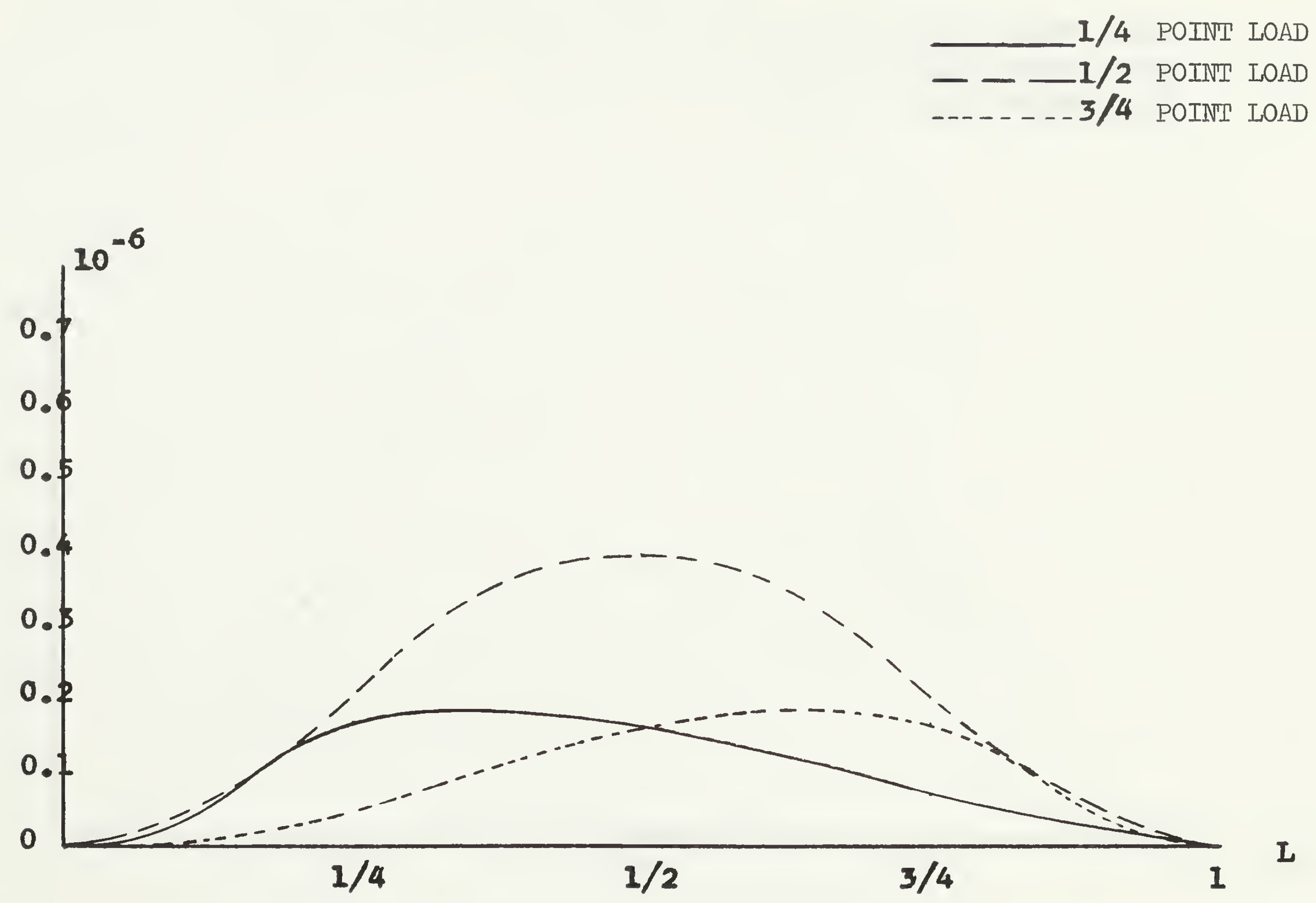


FIG.4-2UNIT LOAD DEFLECTION OF DECK CENTER GIRDER

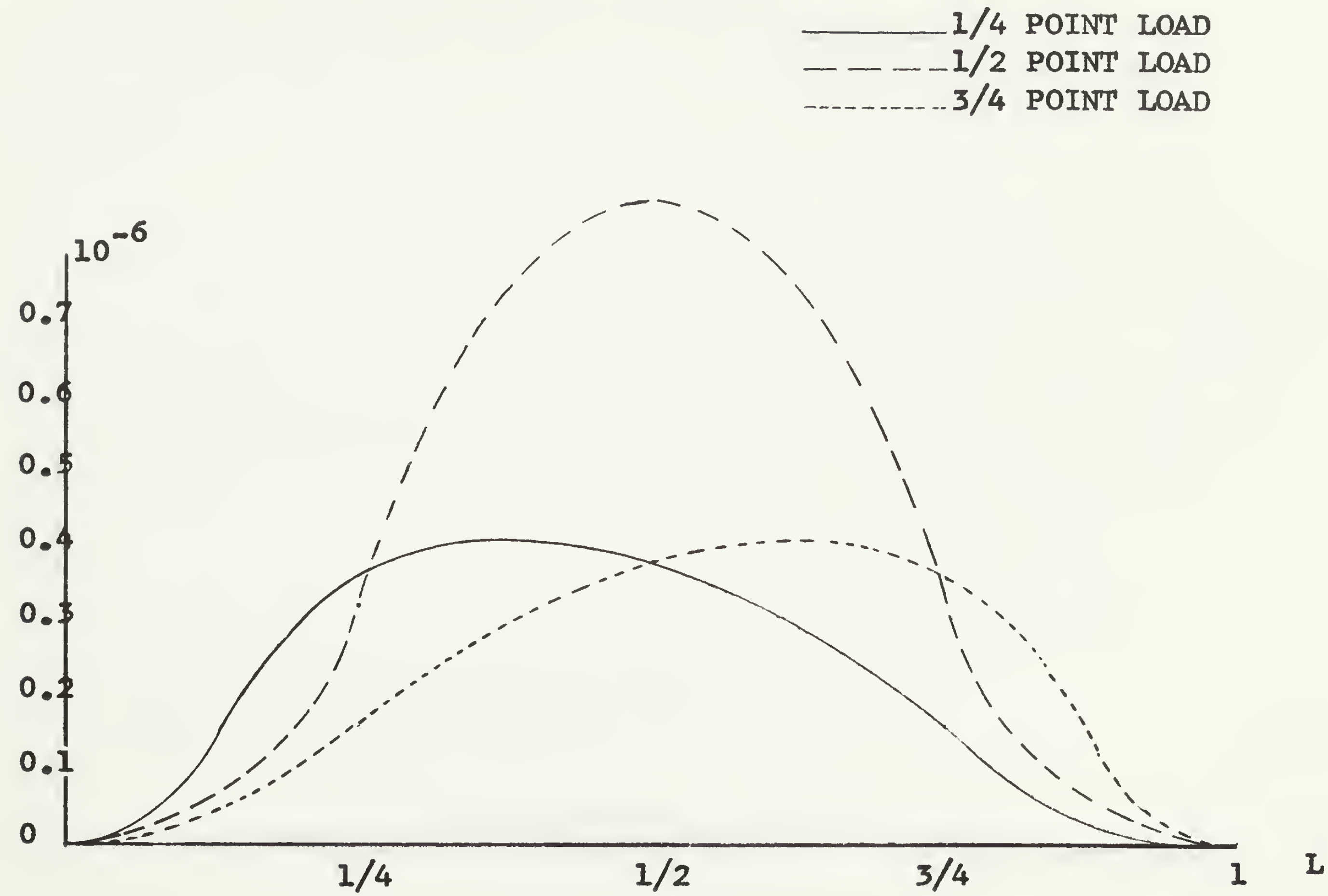
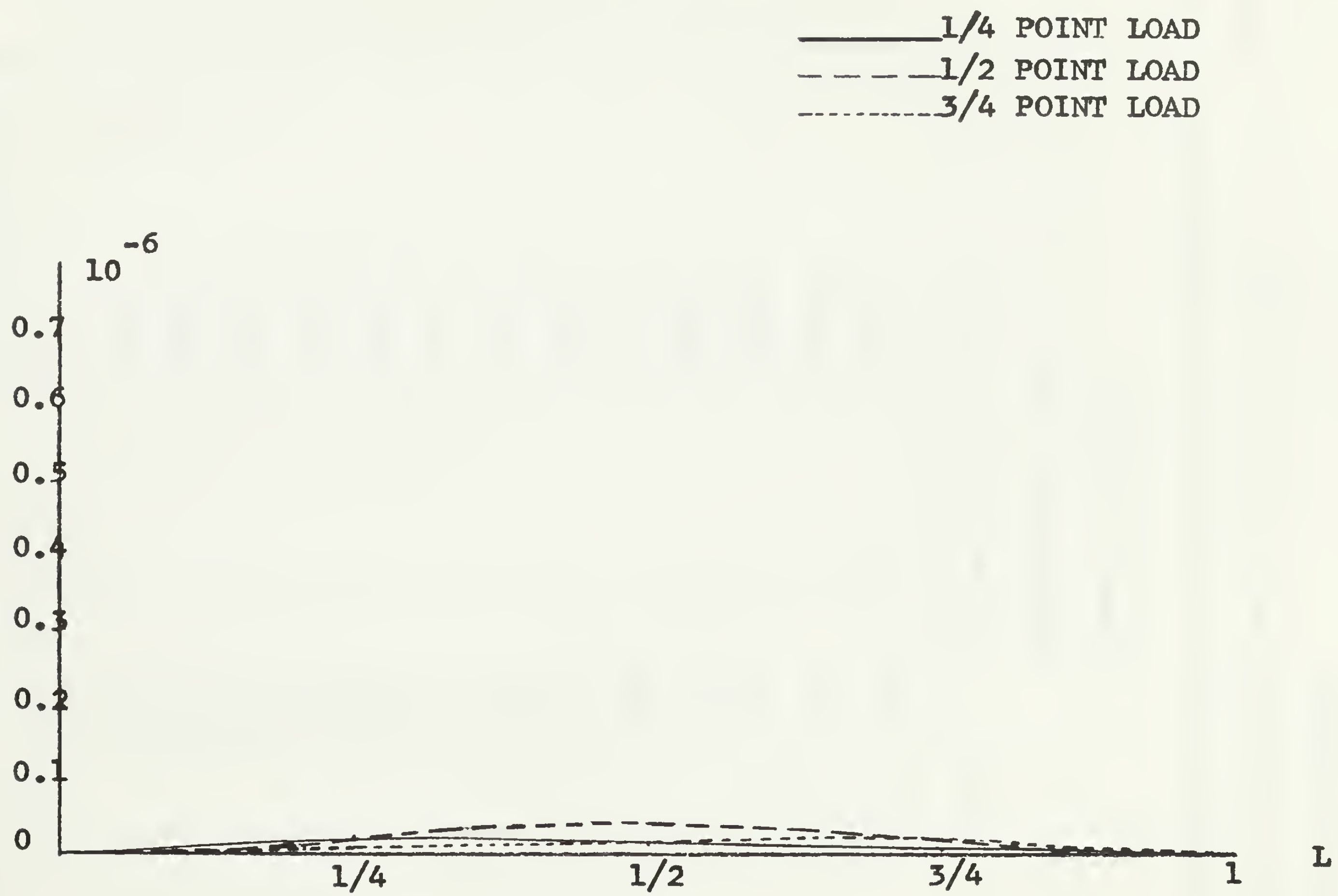


FIG. 4-3 UNIT LOAD DEFLECTION OF LONGITUDINAL BHD.



One of the computed results is tabulated below as Table 5.

X \ Angle	Deflection angle in Radian x 10^{-4}
X(1)	0.475742
X(2)	-0.413502
X(3)	-0.226263
X(4)	0.128071
X(5)	0.171848
X(6)	-0.369586
X(7)	-0.501310
X(8)	0.550122
X(9)	0.647280
X(10)	-0.336503
X(11)	0.028208
X(12)	-0.080296
X(13)	-0.070333
X(14)	0.096957

Table 5. Acquired unknown variables X at the forward frame (load 1) in two dimensional analysis.

From these values we can get moment at edge 1 and 2 (left and right side or upper and lower side) for every span. You can see one of the results in Table 6.

SPAN No.	MOMENT 1 x 10^3	MOMENT 2 x 10^3
1	-0.242163	0.914909
2	-0.894309	1.194515
3	-1.177446	0.960957
4	-0.933445	10.523114
5	0.388150	-0.975332
6	0.936319	-1.158020
7	1.201756	-0.847029
8	0.799642	-11.777653
9	-0.059339	0.242128
10	0.039037	-0.020693
11	-0.043852	-0.170775
12	0.047344	-0.027552
13	12.349345	-10.523304
14	-0.169027	-0.328803
15	-0.283449	-0.571738

Table 6. Moment of the forward frame

From these end moments we can get very easily a middle portion moment by simply superimposing loading to each span. This work can be done very easily if we use the Computor for every span. The results are plotted in figures.

VI. FIXED POINT METHOD

A. General

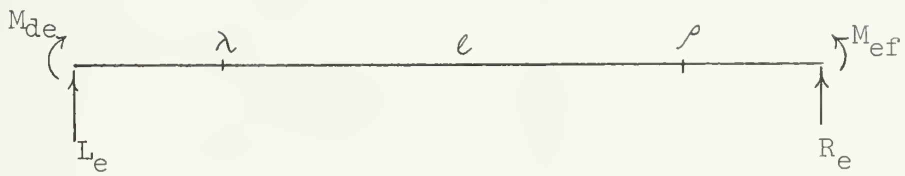
For the purpose of checking above results and investigating shear effect, we will apply here numerical computation by the method of

fixed point. The fixed point method is originated from the three moment equation. In this approach, we have the following assumptions:

(1) All nodal points are fixed in space.

(2) No shear force is considered.

We will not discuss here in detail about this derivation (see Appendix), but will apply this method in practical calculation using the same loading conditions to the same scantling frame ship.



Then the moment at the ends of the span can be expressed by the following formulas.

$$M_{ef}\left(\frac{\rho'_e}{\rho_e} - \frac{\lambda_e}{\lambda'_e}\right) = -\frac{6}{\ell_e'^2}\left(L_e - R_e\frac{\lambda_e}{\lambda'_e}\right)$$

$$M_{de}\left(\frac{\lambda'_e}{\lambda_e} - \frac{\rho_e}{\rho'_e}\right) = -\frac{6}{\ell_e'^2}\left(R_e - L_e\frac{\rho_e}{\rho'_e}\right)$$

Here

$\rho'_e = \ell'_e - \rho_e$

$\lambda'_e = \ell'_e - \lambda_e$

ℓ'_e is the span length

In generally fixed point

$$\lambda_n = \frac{\Sigma \mathcal{E} - \mathcal{E}_n}{6E \frac{I_n}{\ell_n} + 3 \left(\Sigma \mathcal{E} - \mathcal{E}_n \right)} \ell_n$$

$$\rho_n = \frac{\Sigma \omega - \omega_n}{6E \frac{I_n}{\ell_n} + 3 \left(\Sigma \omega - \omega_n \right)} \ell_n$$

and stiffness

$$\mathcal{E}_n = 6E \frac{I_n}{\ell_n} \quad \frac{1 - \frac{\lambda_n}{\ell_n}}{2 - 3 \frac{\lambda_n}{\ell_n}}$$

similarly

$$\omega_n = 6E \frac{I_n}{\ell_n} \quad \frac{1 - \frac{\rho_n}{\ell_n}}{2 - 3 \frac{\rho_n}{\ell_n}}$$

And R_e and L_e can be found out in some handbooks.

That is:

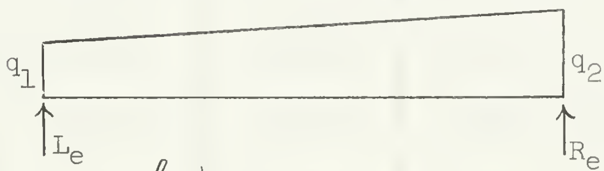
$$L_e = A_o \bar{x}$$

$$R_e = A_o (1 - \bar{x})$$

A_o is the area below the simple moment curve.

and $\bar{x}$ is the distance from the left side to the center of the area A_o .

For our computation purpose we can find out the following data from the handbook.



$$L_e = \frac{\ell_e{}^{,4}}{360} (8q_2 + 7q_1)$$

$$R_e = \frac{\ell_e{}^{,4}}{360} (7q_2 + 8q_1)$$

In generally fixed point

$$\lambda_n = \frac{\Sigma \mathcal{E} - \mathcal{E}_n}{6E \frac{I_n}{\ell_n} + 3\,(\Sigma \mathcal{E} - \mathcal{E}_n)}\,\ell_n$$

$$\rho_n = \frac{\Sigma \omega - \omega_n}{6E \frac{I_n}{\ell_n} + 3\,(\Sigma \omega - \omega_n)}\,\ell_n$$

and stiffness

$$\mathcal{E}_n = 6E \frac{I_n}{\ell_n} \quad \frac{1 - \frac{\lambda_n}{\ell_n}}{2 - 3\frac{\lambda_n}{\ell_n}}$$

similarly

$$\omega_n = 6E \frac{I_n}{\ell_n} \quad \frac{1 - \frac{\rho_n}{\ell_n}}{2 - 3\frac{\rho_n}{\ell_n}}$$

And R_e and L_e can be found out in some handbooks.

That is:

$$L_e = A_o \bar{x}$$

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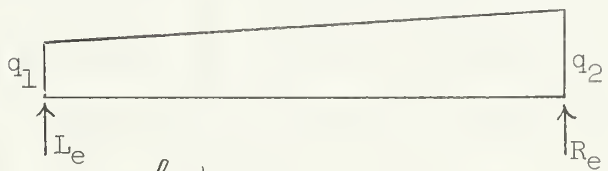
A_o is the area below the simple moment curve.

and $\bar{x}$ is the distance from the left side to the center of the area A_o .

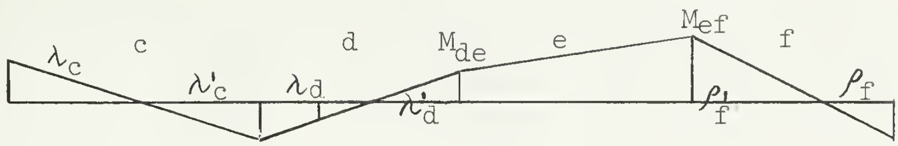
For our computation purpose we can find out the following data from the handbook.



$$L_e = \frac{\ell_e{}' 4}{360}(8q_2 + 7q_1)$$

$$R_e = \frac{\ell_e{}' 4}{360}(7q_2 + 8q_1)$$

If once we get moment for each span, then we can compute each joint moment using stiffness and fixed points by one-by-one adding process as follows:



But in doing so, we must be very careful in the sign for some opening span. (Ref. illustrative calculation of moment distribution Table 12.)

Now here is the practically computed results listed.

	I_{in^4}	ℓ_{in}	$\frac{I_n}{\ell_n}$	Unit $\frac{I_n}{\ell_n} \times \frac{\ell_{15}}{\ell_{15}}$
1	30828.851	280.539	214.955	2.480
2	84741.710	301.006	281.528	3.248
3	45197.227	137.987	327.546	3.779
4	60847.290	137.987	440.963	5.088
5	5282.347	301.006	17.548	0.202
6	25839.827	125.984	205.104	2.366
7	21014.376	125.984	166.801	1.924
8	3656.705	301.006	12.148	0.140
9	25839.827	125.984	205.104	2.366
10	20716.733	125.984	164.439	1.897
11	3083.156	301.006	10.242	0.118
12	30828.851	198.454	155.345	1.792
13	23604.205	198.454	118.940	1.372
14	22181.710	301.006	73.691	0.850
15	24303.928	280.539	86.632	1.000

Table 7

Define $K = 6E \frac{I_{15}}{\ell_{15}}$

In the next table the computed values of λ_n/ℓ_n , ε_n/K , ρ_n/ℓ_n and ω_n/K are shown.

	λ_n/ℓ_n	ε_n/K	ρ_n/ℓ_n	ω_n/K
1	0.232	1.055	0.236	1.022
2	0.219	1.375	0.224	1.382
3	(0.250)0.248	1.419	0.256	1.428
4	0.200	2.159	0.203	2.164
5	0.240	0.813	0.245	0.818
6	0.223	1.107	0.221	1.104
7	(0.250)0.275	1.154	0.277	1.189
8	0.240	(3.816)3.021	0.205	2.920
9	0.327	0.561	0.262	0.516
10	0.327	0.077	0.328	0.077
11	0.327	0.092	0.328	0.092
12	0.327	0.135	0.328	0.133
13	0.260	1.969	0.222	1.893
14	0.266	0.610	0.268	0.612
15	0.284	0.703	0.285	0.703

Table 8

On the next page, ρ/ρ' and λ/λ' tables are also prepared.

	ρ/ρ'	λ/λ'
1	0.308	0.302
2	0.288	0.208
3	0.344	0.329
4	0.254	0.250
5	0.324	0.315
6	0.283	0.287
7	0.383	0.379
8	0.257	0.315
9	0.355	0.485
10	0.424	0.485
11	0.424	0.485
12	0.488	0.485
13	0.285	0.351
14	0.366	0.362
15	0.398	0.396

Table 9-1

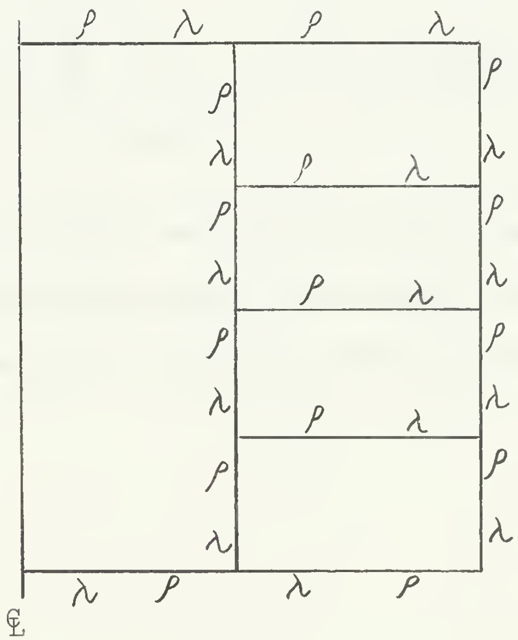


Table 9-2

The computed end moment by its own span load are listed in Table 10. (We restricted our computation in the forward frame only.)

SPAN No.	LOAD I		LOAD II	
	MOMENT 1 x 10^3	MOMENT 2 x 10^3	MOMENT 1 x 10^3	MOMENT 2 x 10^3
1	0.42470	0.52657	0.42470	0.52657
2	0.47568	0.48144	0.47568	0.48144
3	0.81965	0.79450	0.81965	0.79450
4	1.03658	1.03886	1.03658	1.03886
5	-0.44029	-0.54524	0	0
6	-0.46159	-0.49807	0	0
7	-0.87103	-0.89650	0	0
8	-0.95682	-0.13608	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	-7.95302	-5.59423	-7.95302	-5.59423
14	0	0	0	0
15	0	0	-6.82683	-6.89544

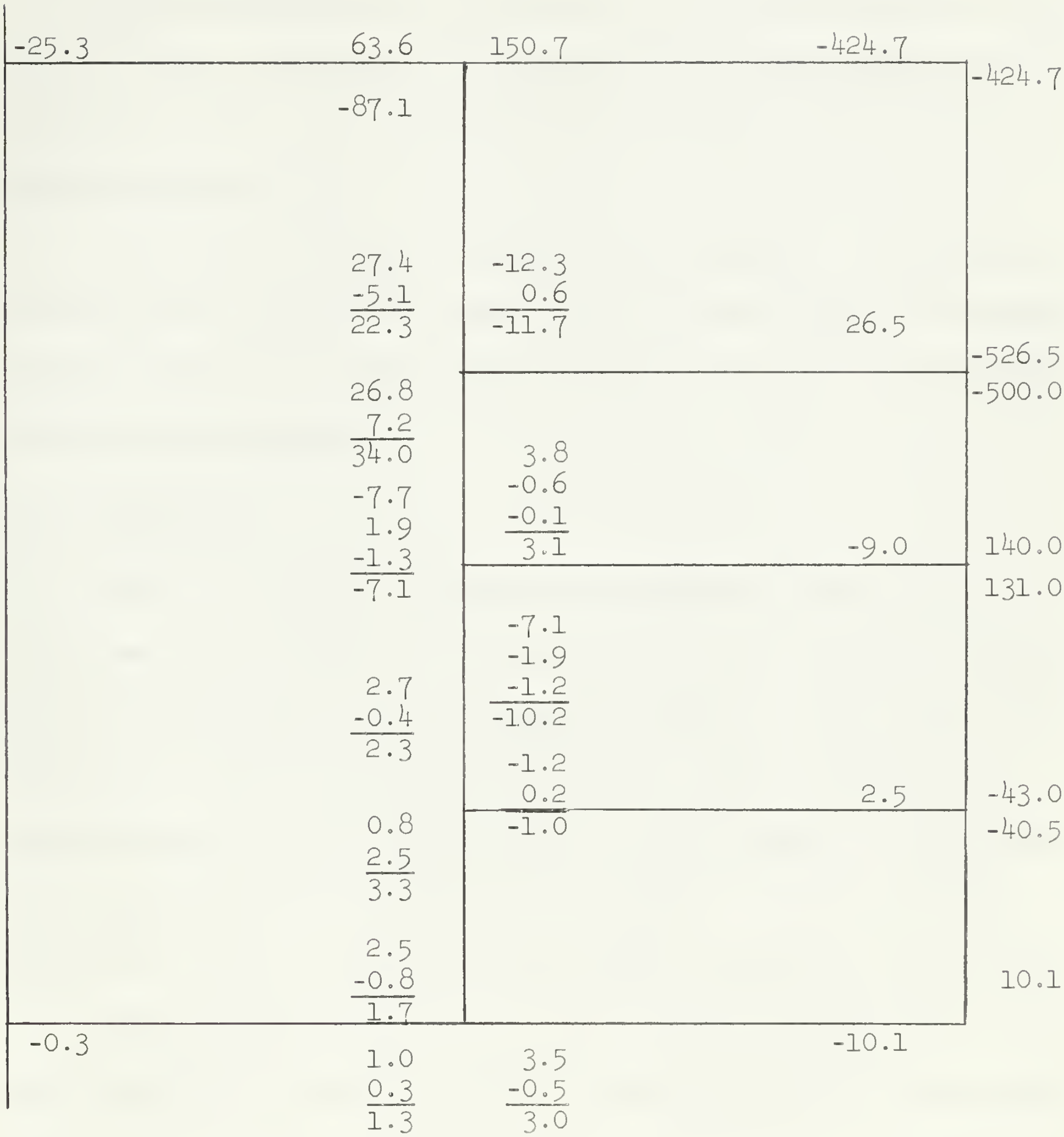
Table 10

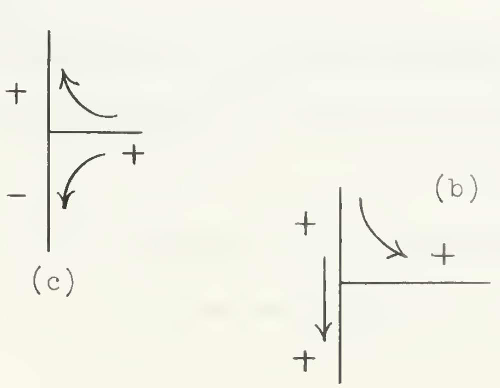
From the above computed end moment at edge 1 and 2 we can distribute moment by using stiffness and fixed points ρ and λ. (See Table 12.) Then we can get the following final end moment.

Table 11. Final end moment in tons-inch

SPAN No.	LOAD I		LOAD II	
	MOMENT 1 x 10^3	MOMENT 2 x 10^3	MOMENT 1 x 10^3	MOMENT 2 x 10^3
1	0.2968	0.7627	0.3396	0.7616
2	0.7425	1.1749	0.7536	1.1315
3	1.2091	0.4132	1.1643	0.5683
4	0.3464	6.2266	0.5379	5.6970
5	-0.3354	-0.7174	0.1312	0.2012
6	-0.6946	-1.3596	0.2248	-0.8502
7	-1.4136	-0.1739	-0.9191	2.4365
8	-0.0980	-7.3683	2.6901	-1.9521
9	0.1419	0.2698	-0.0935	0.3396
10	-0.0228	-0.0202	-0.0236	-0.0080
11	0.0535	0.0342	0.0689	-0.0328
12	-0.0759	-0.0668	-0.2536	-0.0304
13	-8.4853	-6.2266	-10.2938	-5.6970
14	0.0662	-0.1935	0.0049	0.0377
15	0.6203	-1.1170	-6.3110	-8.3417

Table 12. Illustrative calculation of moment distribution for span No. 1 load





Once we get each span's end moment then we can get various point moments very easily by superimposing process. And these numerical computational results are plotted in figures.

Also stress curves are available in this present paper, but we did not compute the moments and stresses for all the frames by the fixed point method. We only calculated the forward frame for both loading conditions.

The reason is this: the fixed point method is, of course, a two dimensional one and each nodal point is fixed in space; therefore, a little difference of water head will bring us a very insignificant computational difference.

VII. DISCUSSION

Most of the naval architects are anxious to learn about the three-dimensional effects on the transverse frame strength of the ship. But the computation is rather lengthy and tedious to work out so the architects estimate the effect on transverse frame strength by the simplified method which satisfies them. In this paper we dealt with this lengthy calculation by use of the computor. It cannot be done without this machine because so many unknown variables are interacted.

Even a casual glance at the graph will show that there are not many differences between the two-dimensional and the three-dimensional analysis.

Especially in load condition 1, the results are nearly the same. Also in load condition 2, we do not see much difference between these two.

In load condition 1, at the forward frame (Fig. 5) we can see also almost no differences between the fixed point method and our

present method in the upper half of the frame that is span No. 1, 2, 5, 6, 9, 10, 11 and 14. That means that lightly loaded spans do not have any shear effect at all, but loading is steadily increased, then the discrepancy is noticeable.

We are now going to examine these graphs individually for more detail.

A. Load Condition 1

(1) This loading condition is ballasted with sea water in the center tank and the side shell is exhibited to the hydrostatic load. This is one of the most favorable loading conditions. In this loading condition, the forward frame does not show us much discrepancy between two-dimensional and three-dimensional analysis. In this loading condition, junction points 1-2 and 2-1; 2-2 and 3-1; 3-2 and 4-1; also 5-2 and 6-1; 6-2 and 7-1; and 7-2 and 8-1 remain in space because of the symetric loading condition. Therefore the fixed point method and our slope-deflection method can be comparable in quantities.

Thus, this means if we neglect shear effect, we will lose almost 4,000 Tons-inch moment at the span edge which is most heavily loaded.

(2) Middle frame: We can see a little difference between these two methods, but not too much. This means that the middle frame is the most effected frame among the three frames because the girders will deflect most at the center.

(3) In the aft frame, we can see almost no effect of the three-dimensional effect on the moment curve. Almost the same as the forward frame.

B. Load Condition 2

(1) We think this loading condition is most critical, but still two-dimensional analysis and three-dimensional analysis have the same aspect. They don't change very much, but here are several interesting aspects being displayed. That is, in our method we fixed joint 1-1 and 9-2 and 4-2 and 13-2 so that as a whole side frame and BHD bent something like a semi-fixed beam at the bottom and deck, but by the fixed point method nodal points are fixed in space so that they have almost the same aspect as in load condition 1. But from this curve we can see also a big difference in heavily loaded lower spans.

(2) In the middle frame we can see some conceivable discrepancies between those two methods, but still in good shape; in three-dimensional analysis, however, the moment at point 15-1 and 9-2 is reversed in direction because of the restraints by girders released at 15-1 and 13-1 points. The other spans have almost the same results, nevertheless one can see a little higher moment in the frames 9, 10, 11, 12 and 14.

(3) In the aft frame, the aspect is almost the same as in the forward frame. We cannot see any noticeable discrepancies between those two approaches.

CONCLUSION

Most designers of super tankers are worried about the three-dimensional effect on the transverse frame strength. But from our computation we cannot find large discrepancies between two-dimensional and three-dimensional analysis. However, in the three-dimensional analysis, calculation is more detailed and tedious than in the two-dimensional.

For our present discussion, three-dimensional computation is more academic than practical, otherwise we would not use any simplified design process such as Professor Schade's Design Curves for Cross Stiffened Plating.

Thus our conclusion is: forget about the three-dimensional analysis, unless you intend to use any simplified design process prepared for the three-dimensional analysis; but take into account the shear effect on the two-dimensional analysis on super tanker designs.

FIG. 5 FORWARD FRAME MOMENT CURVE (LOAD 1)

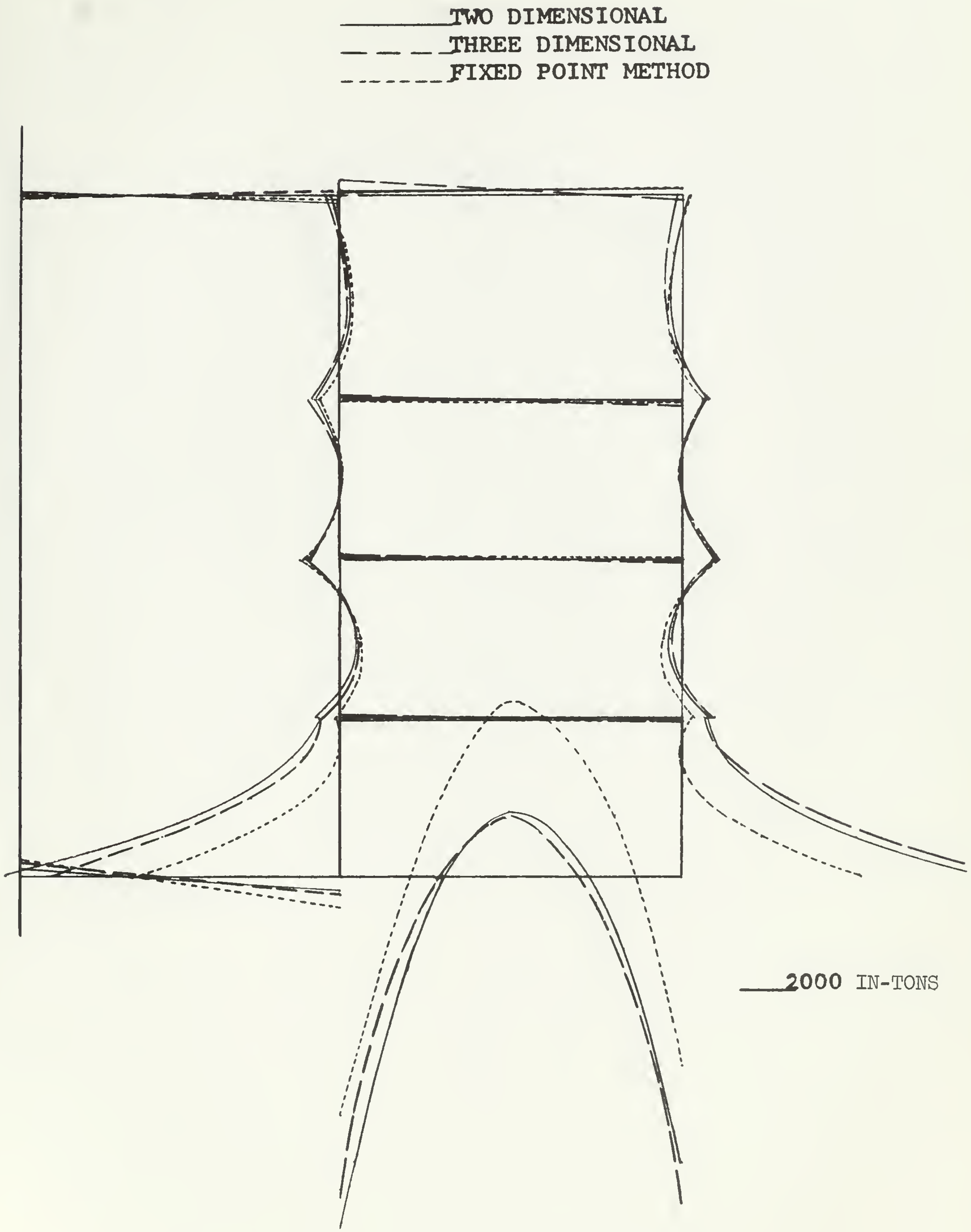


FIG. 6 MIDDLE FRAME MOMENT CURVE (LOAD 1)

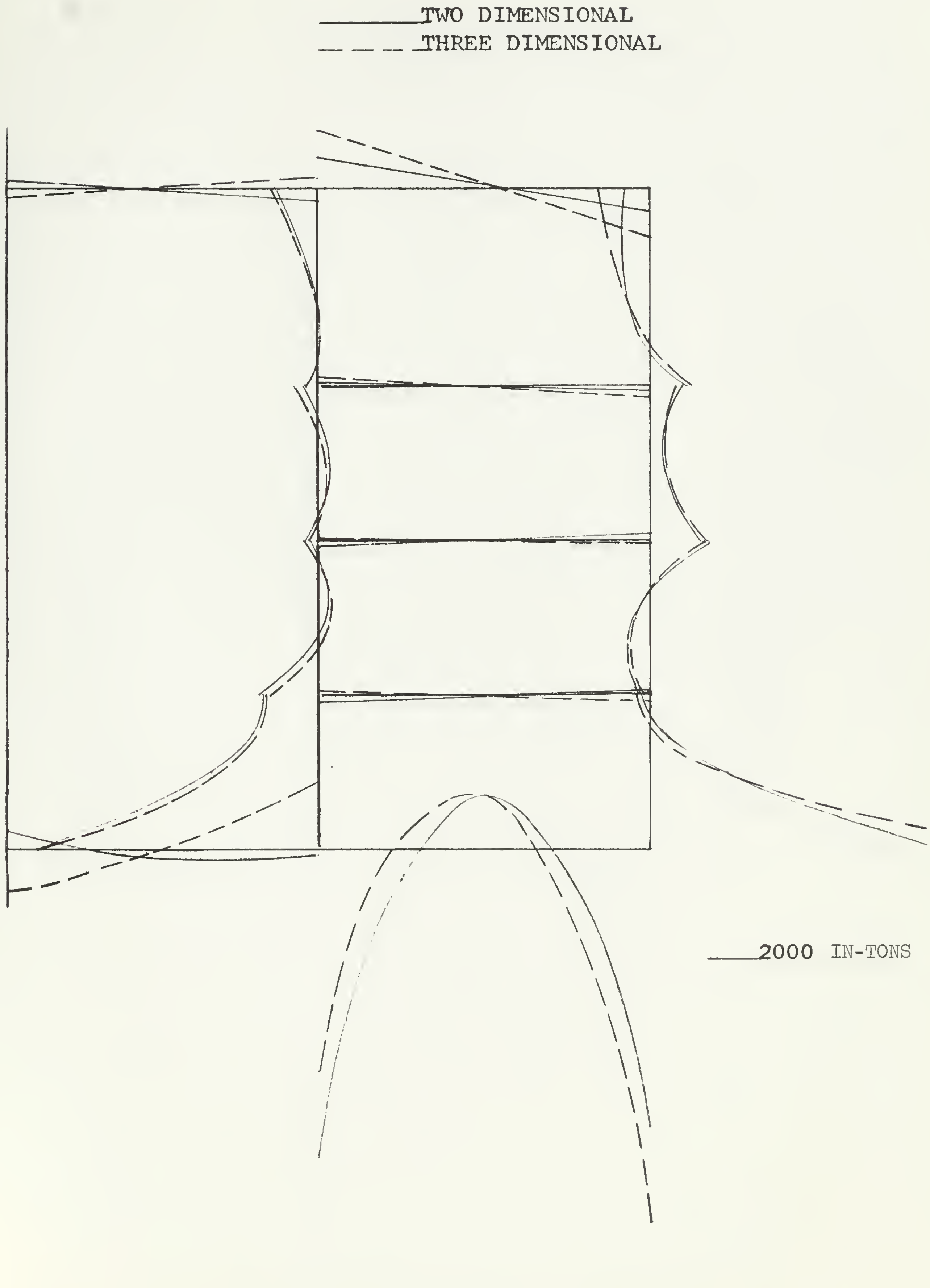


FIG. 7 AFT FRAME MOMENT CURVE (LOAD 1)

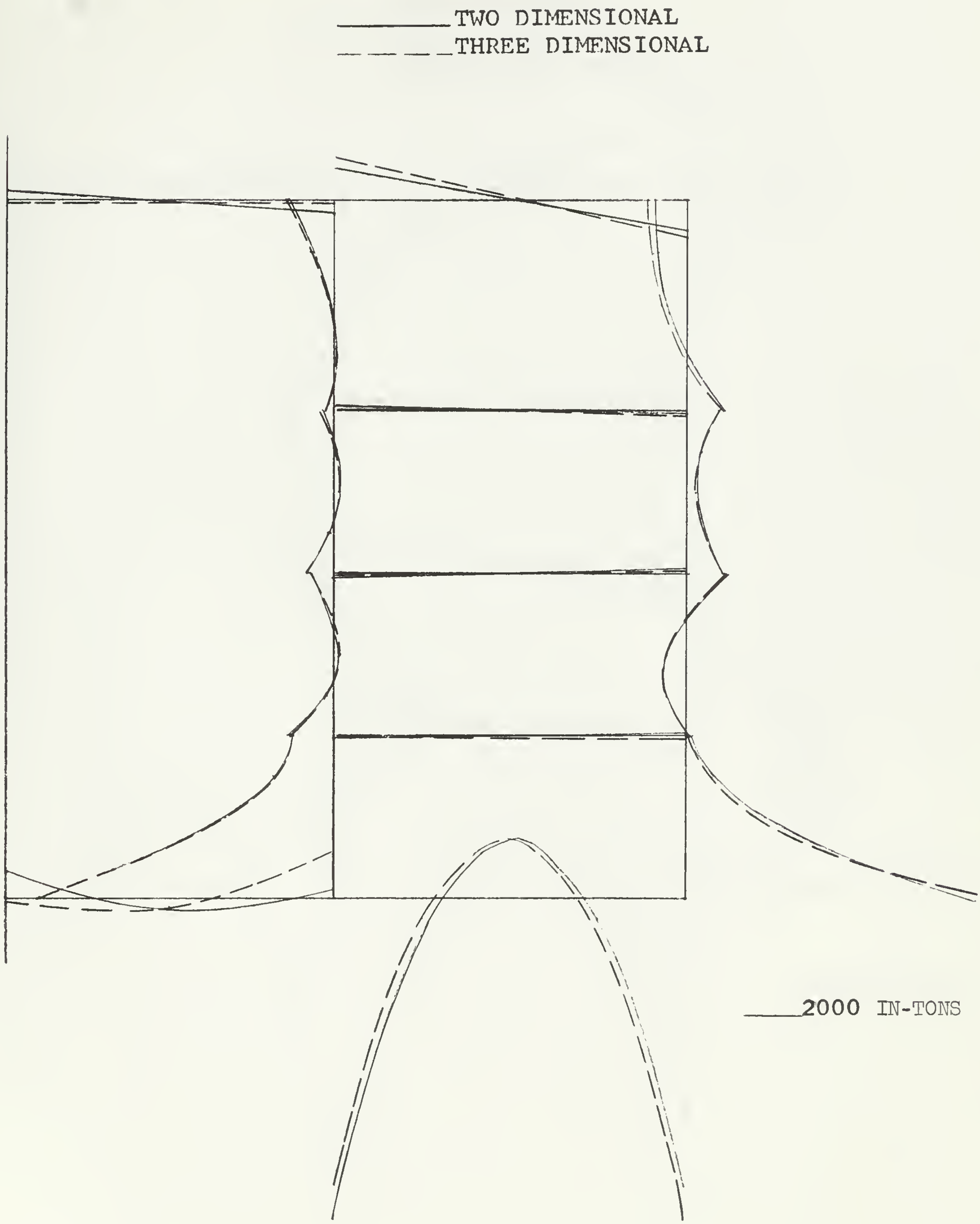


FIG. 8 FORWARD FRAME MOMENT CURVE (LOAD 2)

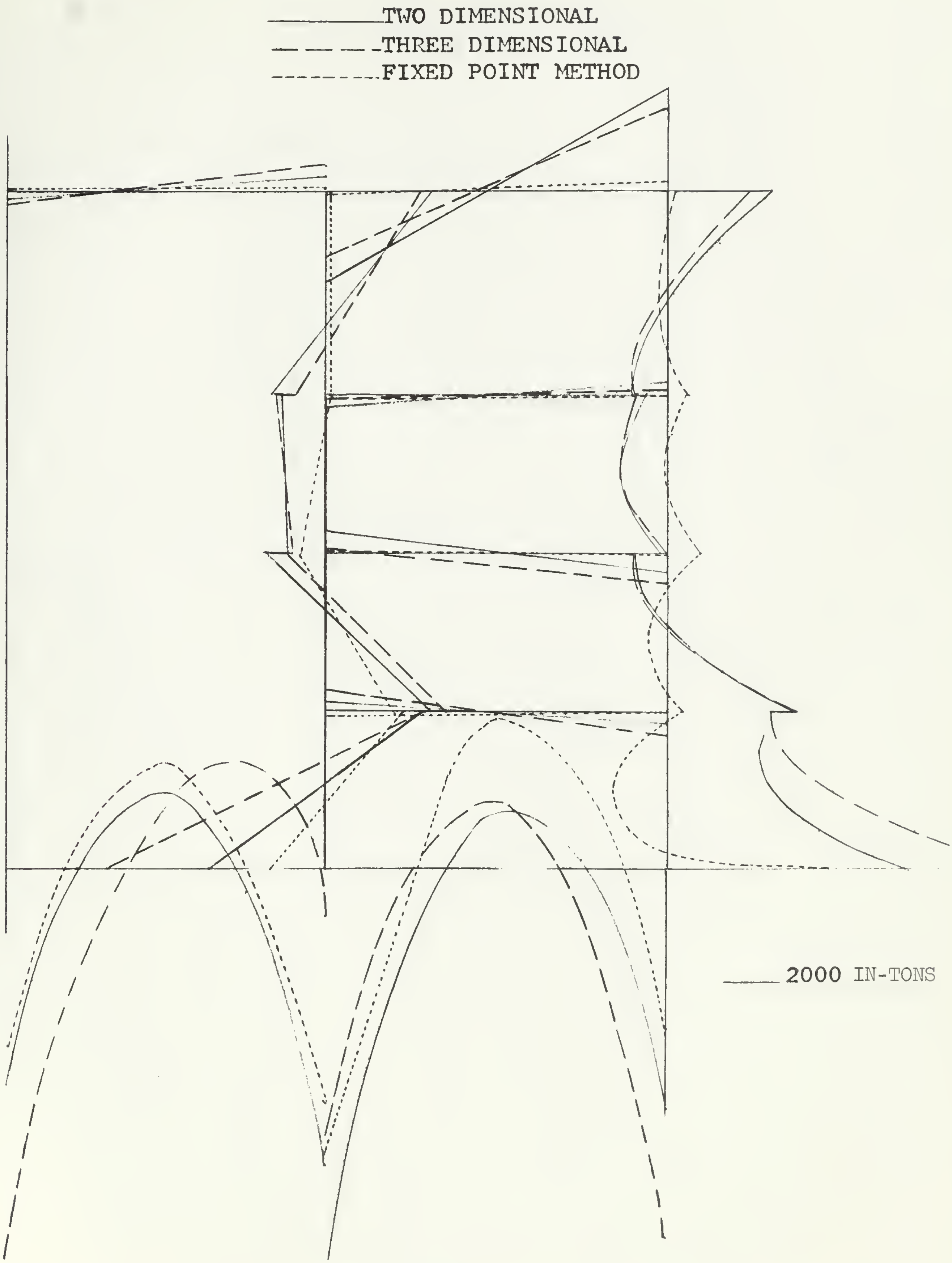


FIG. 9 MIDDLE FRAME MOMENT CURVE (LOAD 2)

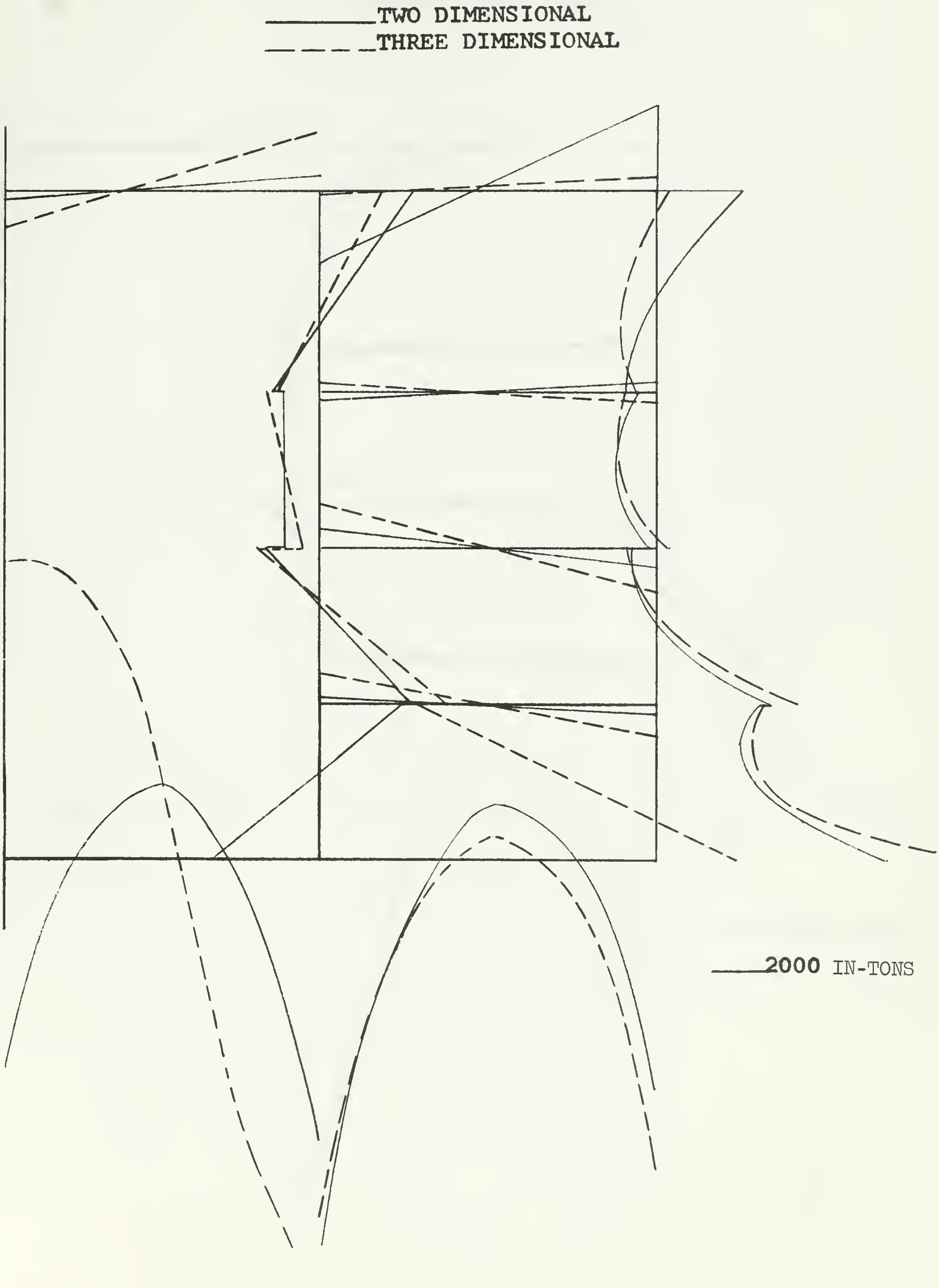


FIG.10 AFT FRAME MOMENT CURVE (LOAD 2)

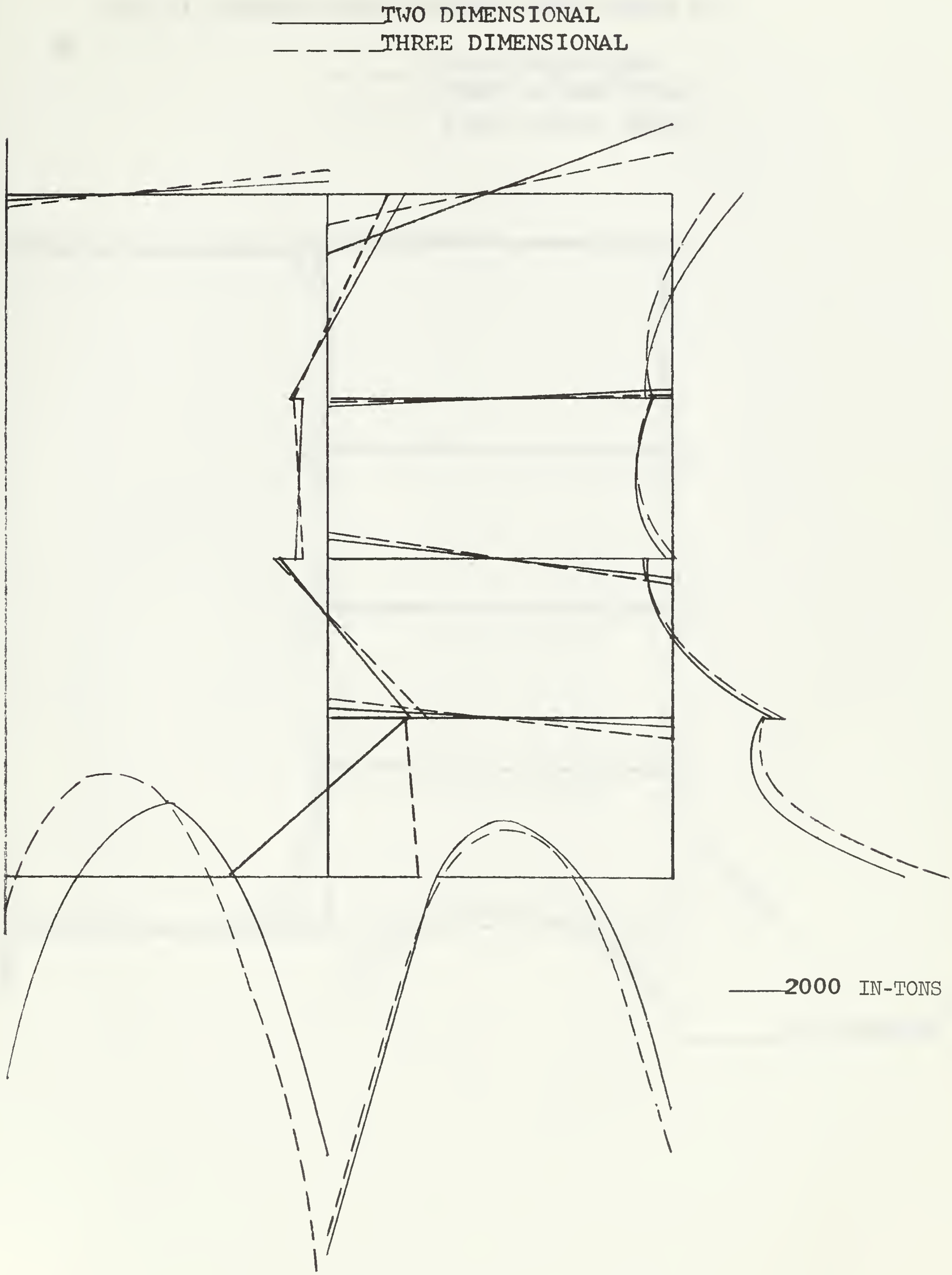


FIG. 11 FORWARD FRAME STRESS CURVE (LOAD 1)

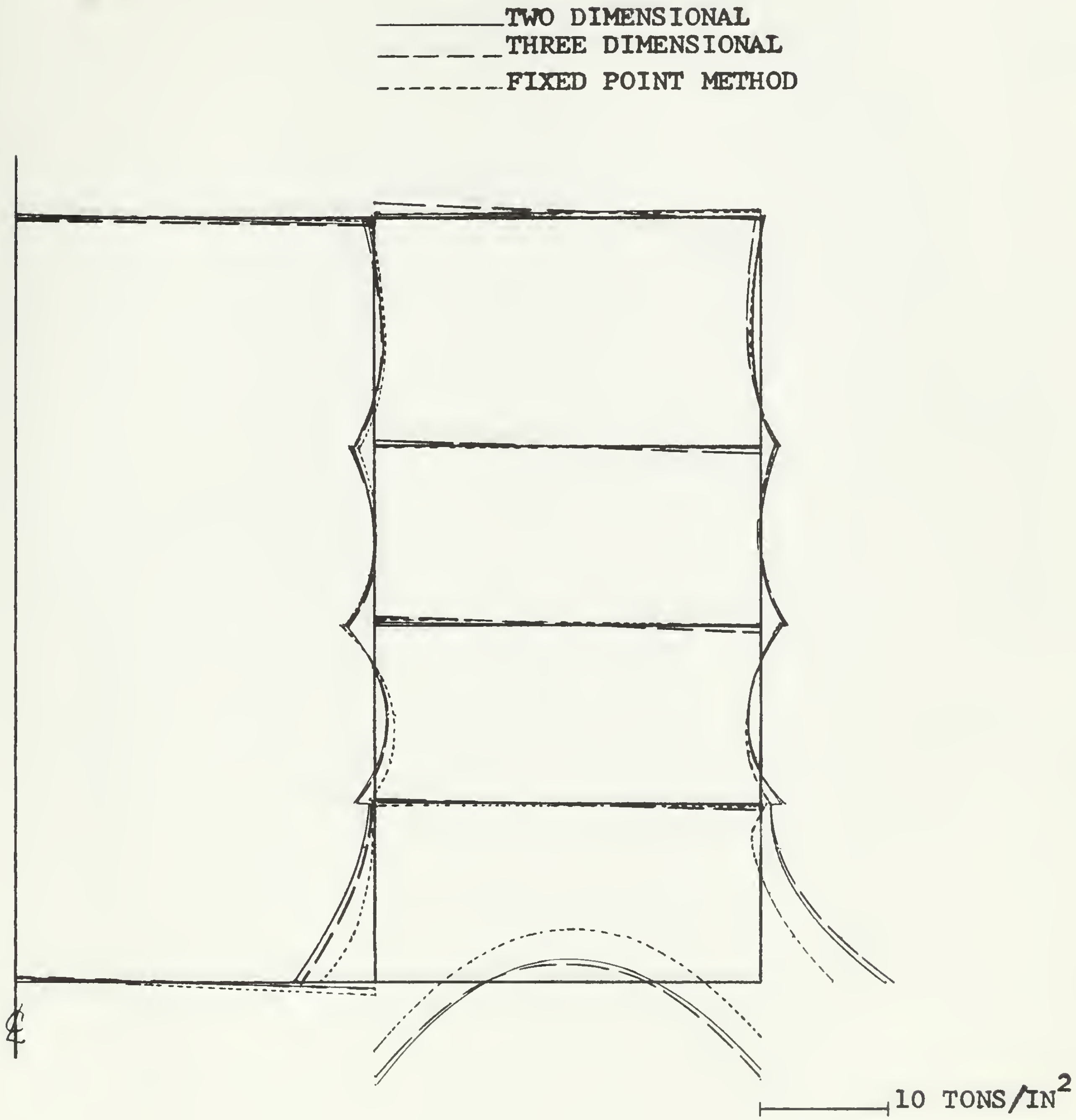


FIG. 12 MIDDLE FRAME STRESS CURVE (LOAD 1)

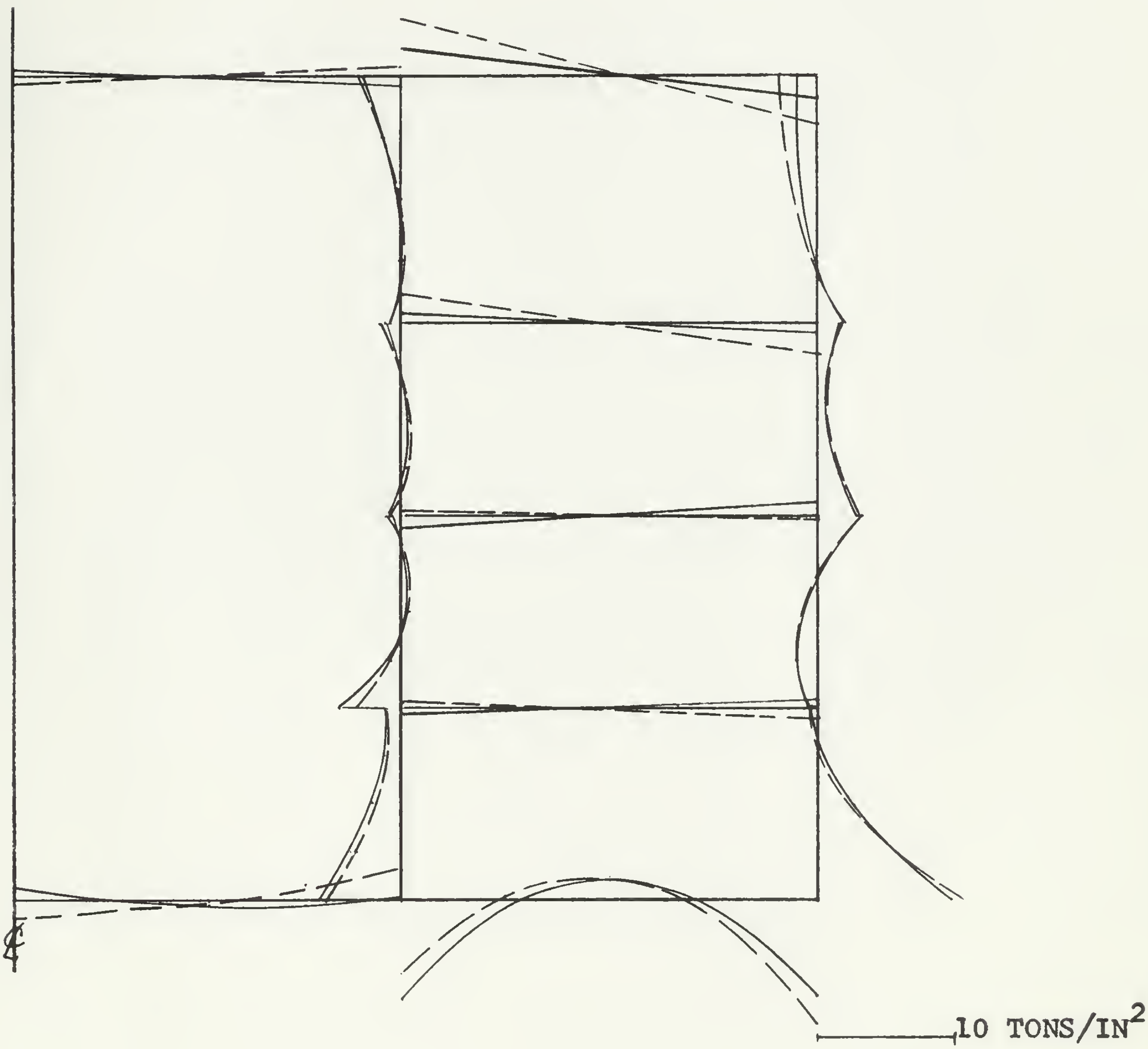


FIG. 13 AFT FRAME STRESS CURVE (LOAD 1)

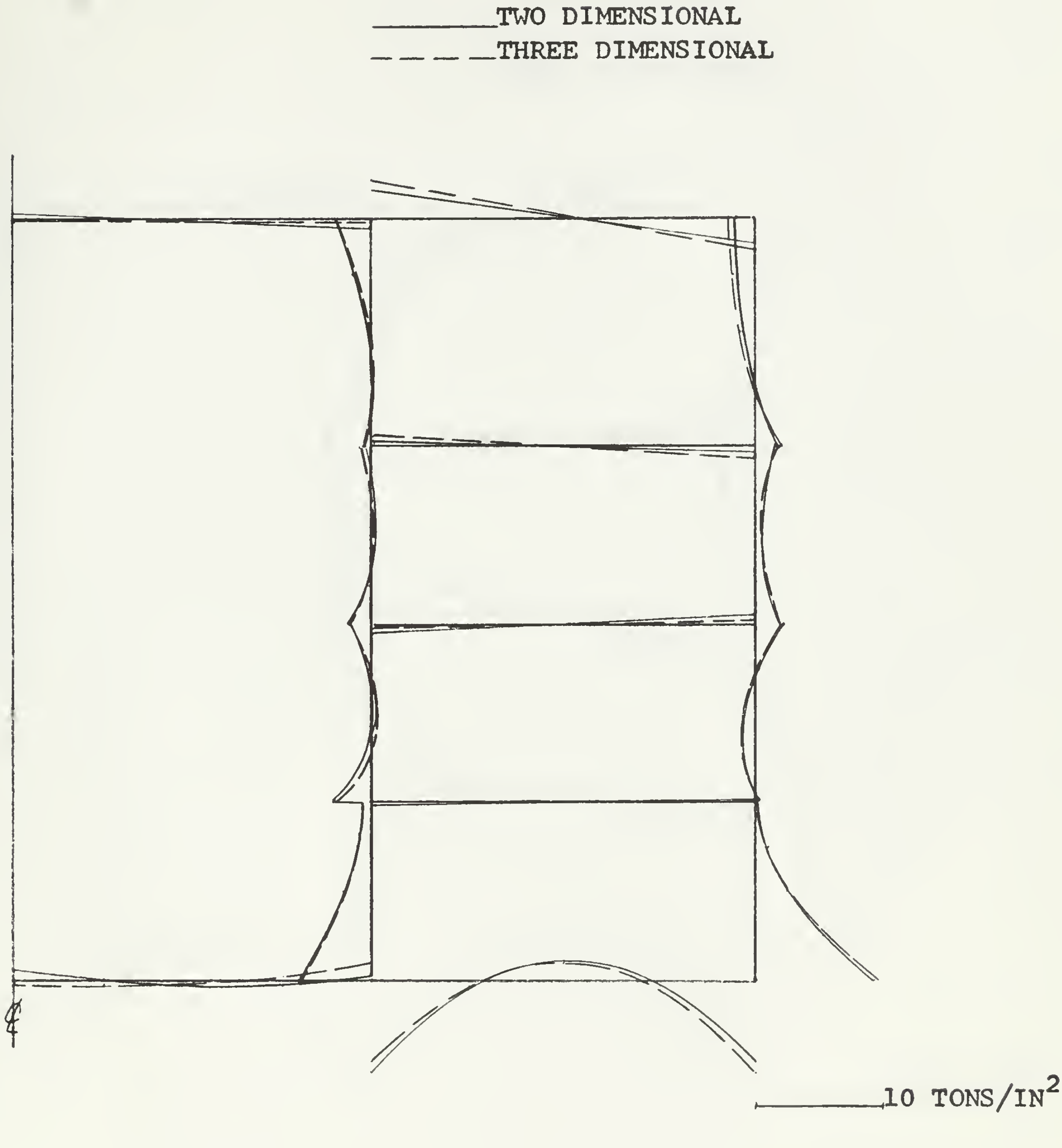
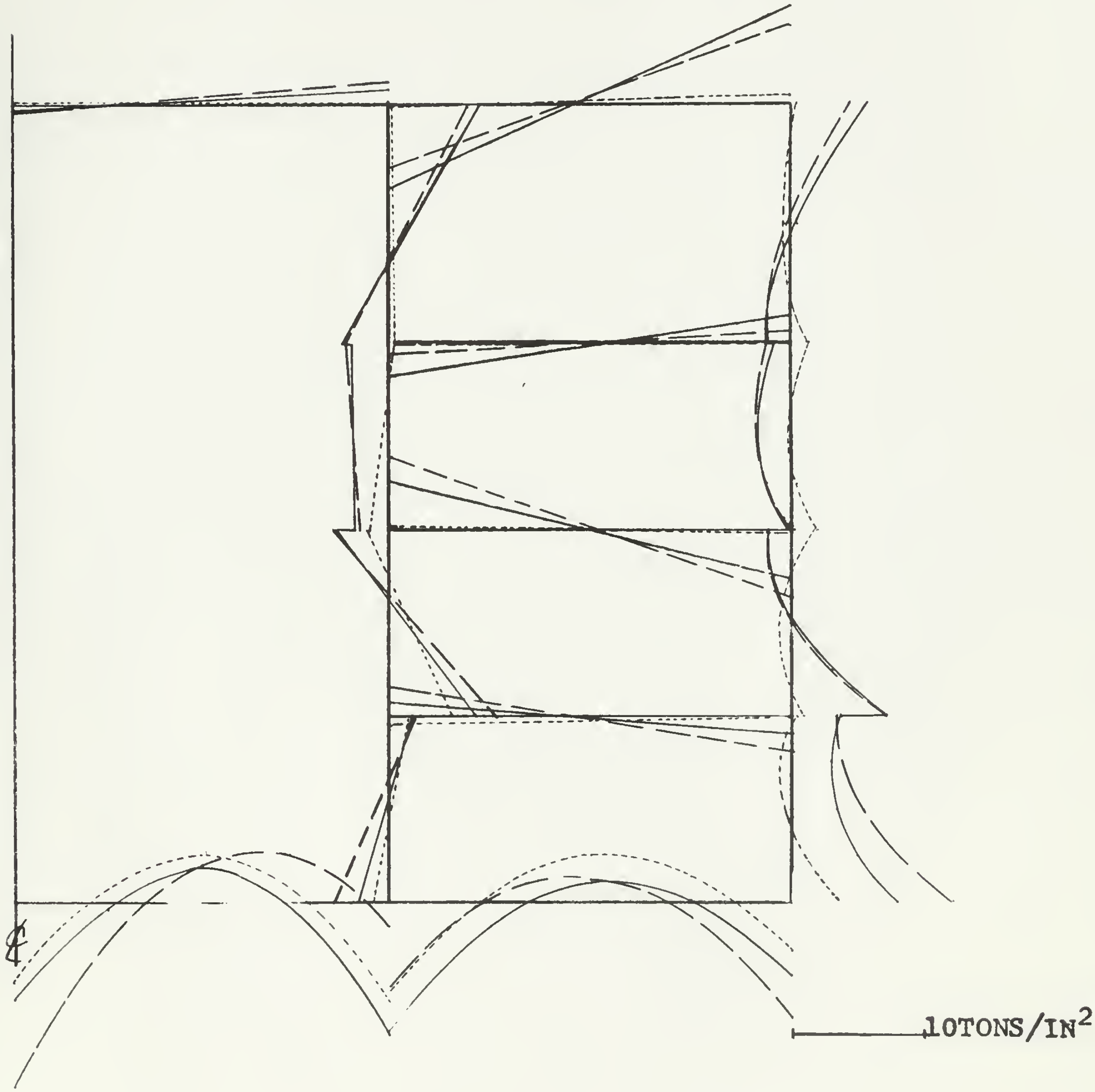


FIG. 14 FORWARD FRAME STRESS CURVE (LOAD 2)

________TWO DIMENSIONAL
_ _ _ _THREE DIMENSIONAL
\-\-\-\-\-\-\-\-FIXED POINT METHOD



[illegible]

[illegible]

FIG. I5 MIDDLE FRAME STRESS CURVE (LOAD 2)

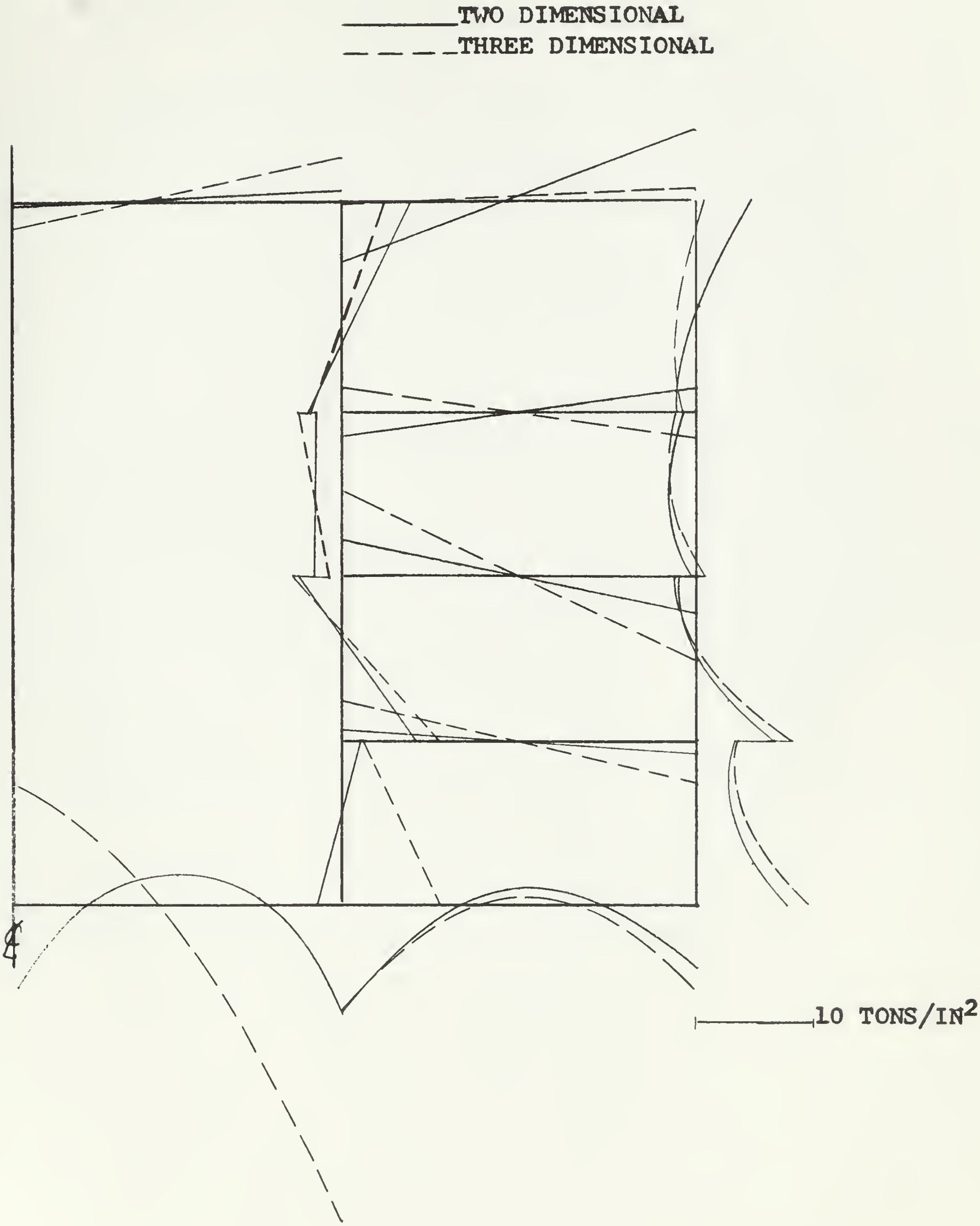
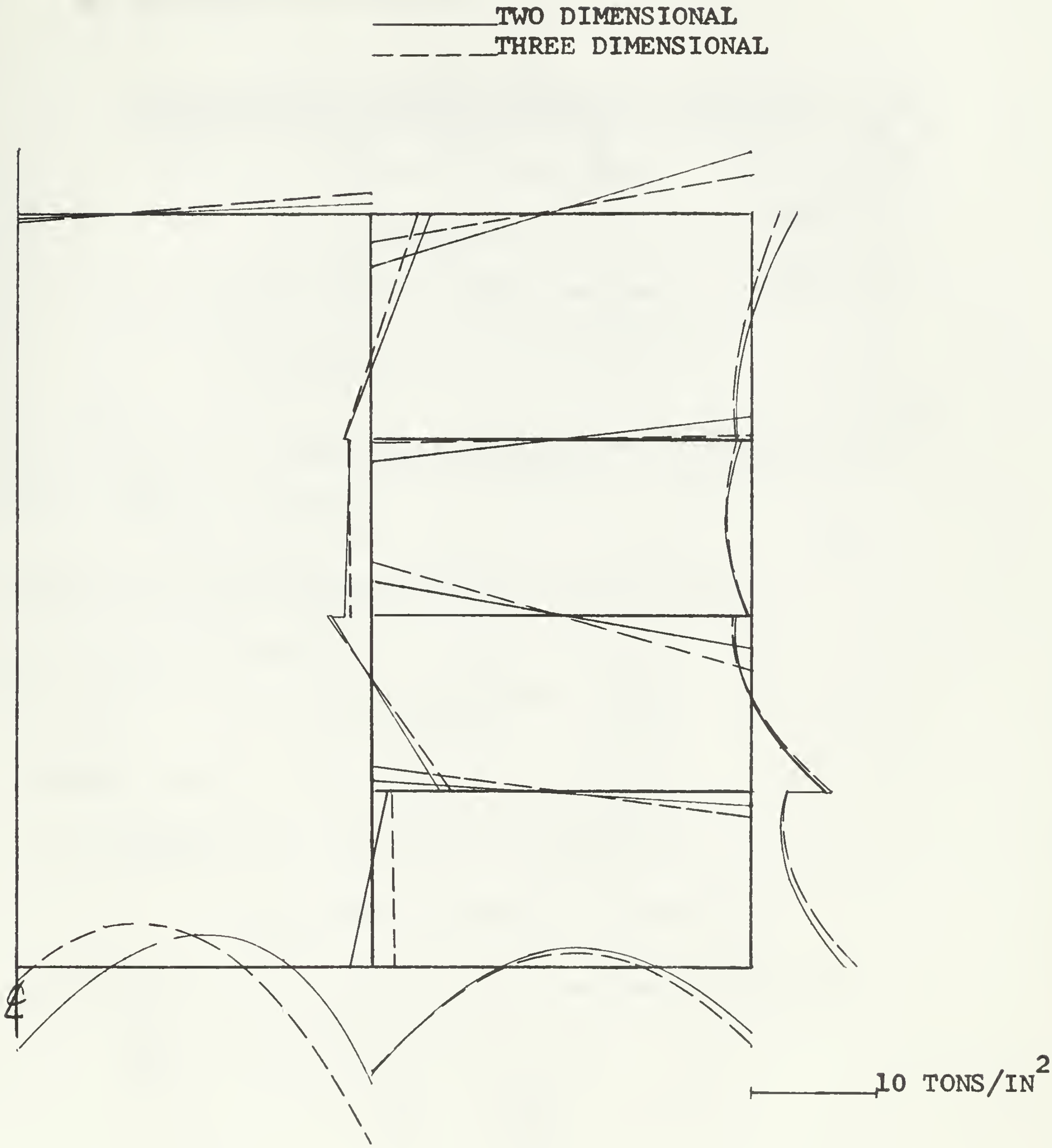
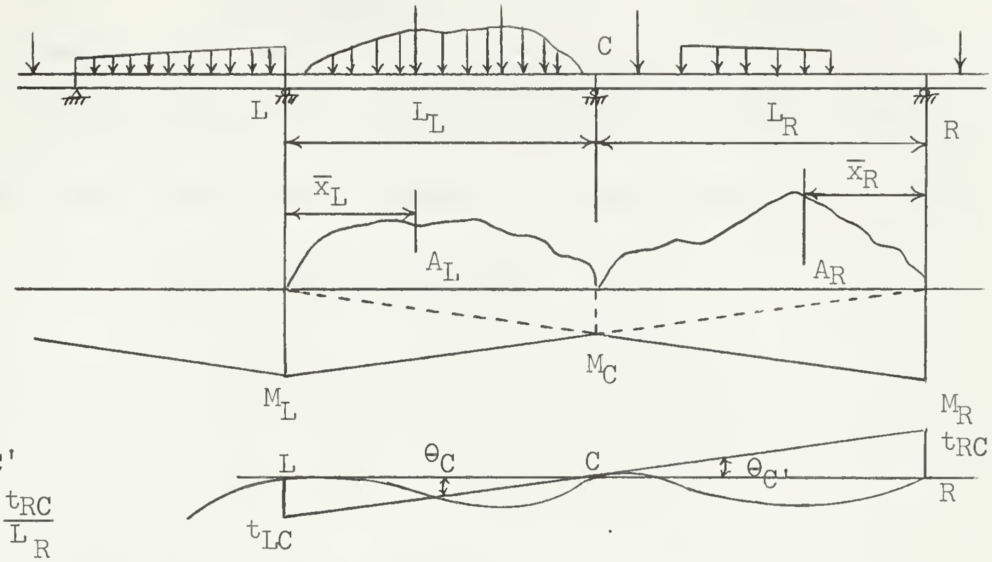


FIG. 16 AFT FRAME STRESS CURVE (LOAD 2)



IX. APPENDIX

1. The three moment equation



$$\theta_C = \theta_{C'}$$

or $$\frac{t_{LC}}{L_L} = -\frac{t_{RC}}{L_R}$$

Then by using the second moment area theorem to obtain

t_{LC} and t_{RC}

$$t_{LC} = \frac{1}{EI_C}\int Mxdx$$

Therefore simply

$$\frac{1}{L_L}\,\frac{1}{EI_L}\left(A_L\bar{x}_L + \frac{1}{2}L_L M_L \frac{1}{3}L_L + \frac{1}{2}L_L M_C \frac{2}{3}L_L\right)$$

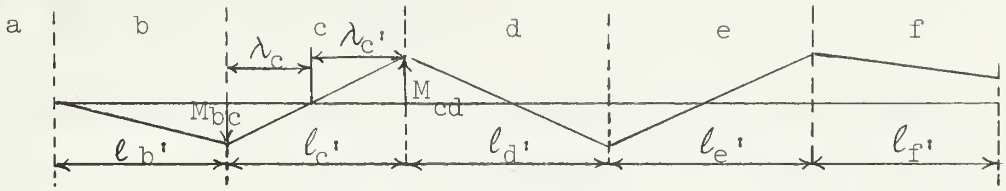
$$= -\frac{1}{L_R}\,\frac{1}{EI_R}\left(A_R\bar{x}_R + \frac{1}{2}L_R M_R \frac{1}{3}L_R + \frac{1}{2}L_R M_C \frac{2}{3}L_R\right)$$

Simplifying the above expression, the three moment equation is:

$$L_L M_L + 2\left(L_L + \frac{I_L}{I_R}L_R\right)M_C + \frac{I_L}{I_R}L_R M_R$$

$$= -6A_L\frac{\bar{x}_L}{L_L} - 6A_R\frac{I_L\bar{x}_R}{I_R L_R}$$

2. Fixed point method



By three moment theory (Ref. Appendix 1) first three support (from left)

$$0 + 2M_{bc}\left(\frac{\ell_b'}{I_b} + \frac{\ell_c'}{I_c}\right) + M_{cd}\frac{\ell_c'}{I_c} = 0$$

(where $A_L = A_R = 0$)

set $\ell_b = \ell_b' \dfrac{1}{I_b}$, etc.

Then

$$0 + 2M_{bc}(\ell_b + \ell_c) + M_{cd}\,\ell_c = 0$$

$$\frac{M_{bc}}{M_{cd}} = -\frac{\ell_c}{2(\ell_b + \ell_c)}$$

By geometry

$$\frac{M_{bc}}{M_{cd}} = -\frac{\lambda_c}{\lambda_c'}$$

Therefore finally

$$\frac{\lambda_c}{\lambda_c'} = -\frac{\ell_c}{2(\ell_b + \ell_c)}$$

Apply three moment equation to span C and D

$$M_{bc}\ell_c + 2M_{cd}(\ell_c + \ell_d) + M_{de}\,\ell_d = 0$$

using relationship

$$\frac{M_{bc}}{M_{cd}} = -\frac{\lambda_c}{\lambda_c'}$$

$$M_{cd}\left(2\ell_c + 2\ell_d - \frac{\lambda_c}{\lambda_c'}\ell_c\right) + M_{de}\,\ell_d = 0$$

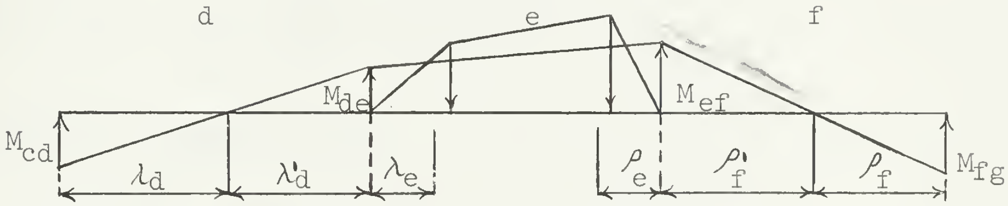
$$\therefore \quad \frac{\lambda_d}{\lambda_d'} = \frac{\ell_d}{2\ell_c + 2\ell_d - \frac{\lambda_c}{\lambda_c'}\ell_c}$$

In general

$$\frac{\lambda_n}{\lambda'_n} \qquad \frac{\ell_n}{2\ell_{n-1} + 2\ell_n - \frac{\lambda_{n-1}}{\lambda'_{n-1}}\ell_{n-1}}$$

NOTE: The position of zero moment is a function of geometry and the degree of fixity at the ends and independent of the type of loads or the magnitude of loads.

LOADED SPAN E



Apply three moment equation to span D and E (D is unloaded, E is loaded).

$$M_{cd}\ell_d + 2M_{de}\,(\ell_d + \ell_e) + M_{ef}\ell_e + 6\frac{R_e}{\ell_e'^2}\,\ell_e = 0$$

Where R_e = Moment of field load moment area about right support of span E

L_e = Moment of field load moment area about left support of span E

$$M_{de}\ell_e + 2M_{ef}\,(\ell_o + \ell_f) + M_{fg}\ell_f + 6\,\frac{L_e}{\ell_e'^2}\,\ell_e = 0$$

Eliminate M_{cd} and M_{fg} by using:

$$M_{cd} = -\frac{\lambda_d}{\lambda'_d}\,M_d\,\ell$$

$$M_{fg} = M_{ef}\,\left(-\frac{\rho_f}{\rho'_f}\right)$$

Then we obtain

$$M_{de}\left(2\ell_d + \ell_e - \frac{\lambda_d}{\lambda'_d}\ell_d\right) + M_{ef}\ell_e + 6\frac{R_e}{\ell_e'^2}\ell_e = 0$$

$$M_{ef}\left(2\ell_e + 2\ell_f - \frac{\rho_f}{\rho'_f}\ell_f\right) + M_{de}\ell_e + 6\frac{L_e}{\ell_e'^2}\ell_e = 0$$

from general equation

$$2\ell_d + 2\ell_e - \frac{\lambda_d}{\lambda'_d}\ell_d = \frac{\lambda'_e}{\lambda_e}\ell_e$$

$$2\ell_e + 2\ell_f - \frac{\rho_f}{\rho'_f}\ell_f = \frac{\rho'_e}{\rho_e}\ell_e$$

Then above equation becomes:

$$M_{de}\frac{\lambda'_e}{\lambda_e} + M_{ef} + 6\frac{R_e}{\ell_e'^2} = 0$$

$$M_{ef}\frac{\rho'_e}{\rho_e} + M_{de} + 6\frac{L_e}{\ell_e'^2} = 0$$

Finally we get

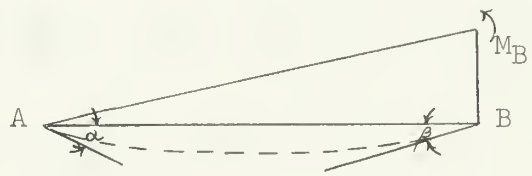
$$M_{ef}\left(\frac{\rho'_e}{\rho_e} - \frac{\lambda_e}{\lambda'_e}\right) = -\frac{6}{\ell_e'^2}\left(L_e - R_e\frac{\lambda_e}{\lambda'_e}\right)$$

$$M_{de}\left(\frac{\lambda'_e}{\lambda_e} - \frac{\rho_e}{\rho'_e}\right) = -\frac{6}{\ell_e'^2}\left(R_e - L_e\frac{\rho_e}{\rho'_e}\right)$$

3. Stiffness factor

Stiffness factor is defined as the reciprocal of the angle of rotation caused by a unit moment.

From the elementary mechanics of materials we know the following relation between angle of rotation and moments



$$\alpha = \frac{M_B}{EI}\frac{L}{6} \qquad \beta = \frac{M_B}{EI}\frac{L}{3}$$

By these relations we can easily obtain

$$\tan\phi_{CD} = \frac{\ell_c}{3EI_c} M_{CD} + \frac{\ell_c}{6EI_c} M_{BC}$$

$$M_{CD} = 1 \,, \quad M_{BC} = -\frac{\lambda_c}{\lambda'_c} M_{CD}$$

$$\tan\phi_{CD} = \frac{\ell_c}{3EI_c} - \frac{\lambda_c}{\lambda'_c}\frac{\ell_c}{6EI_c}$$

If we use the notation $\mathcal{E}_c$ to denote the stiffness of the left hand side member, and substitute λ'_c by $(\ell_c - \lambda_c)$ we will get

$$\mathcal{E}_c = \frac{1}{\tan\phi_{CD}} = \frac{1}{\dfrac{2\ell_c\lambda'_c - \lambda_c\ell_c}{6EI_c(\ell_c - \lambda_c)}} = \frac{1}{\dfrac{2\ell_c(\ell_c - \lambda_c) - \lambda_c\ell_c}{6EI_c(\ell_c - \lambda_c)}}$$

$$= \frac{6EI_c}{\ell_c}\,\frac{\ell_c - \lambda_c}{2\ell_c - 3\lambda_c}$$

In general, the stiffness of the left hand member at the right hand support $\mathcal{E}_n$

$$\mathcal{E}_n = 6E\frac{I_n}{\ell_n}\,\frac{1 - \dfrac{\lambda_n}{\ell_n}}{2 - \dfrac{3\lambda_n}{\ell_n}}$$

Similarly we can express in terms of right hand member

$$-\tan\phi_{CD} = \frac{\ell_d}{3EI_d} - \frac{\lambda_d'}{\lambda_d}\,\frac{\ell_d}{6EI_d} = \frac{2\ell_d\lambda_d - (\ell_d - \lambda_d)\ell_d}{6EI_d\lambda_d}$$

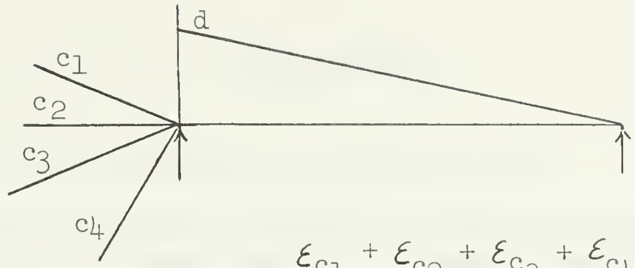
$$= \frac{\ell_d}{6EI_d}\,\frac{2\ell_d - 3\ell_d + 3\lambda_d}{\lambda_d}$$

Therefore

$$\frac{1}{\mathcal{E}_c} = \frac{\ell_d}{6EI_d}\left(-\frac{\ell_d}{\lambda_d} + 3\right)$$

$$\lambda_d = \frac{\mathcal{E}_c}{6E\dfrac{I_d}{\ell_d} + 3\mathcal{E}_c}\,\ell_d$$

In general



$$\lambda_d = \frac{\varepsilon_{c_1} + \varepsilon_{c_2} + \varepsilon_{c_3} + \varepsilon_{c_4}}{6E \frac{I_d}{\ell_d} + 3(\varepsilon_{c_1} + \varepsilon_{c_2} + \varepsilon_{c_3} + \varepsilon_{c_4})} \, d$$

If we use the notation $\Sigma\varepsilon$ including left and right hand members' stiffness, then we have

$$\lambda_n = \frac{\Sigma\varepsilon - \varepsilon_n}{6E \frac{I_n}{\ell_n} + 3(\Sigma\varepsilon - \varepsilon_n)} \, \ell_n$$

Similarly we can apply the above formula from right side to left side. The results are:

$$\omega_n = 6E \frac{I_n}{\ell_n} \frac{1 - \frac{\rho_n}{\ell_n}}{2 - \frac{3\rho_n}{\ell_n}}$$

$$\rho_n = \frac{\omega - \omega_n}{6E \frac{I_n}{\ell_n} + 3(\Sigma\omega - \omega_n)} \, \ell_n$$

$$\varepsilon_n = 6E \frac{I_n}{\ell_n} \frac{1 - \frac{\lambda_n}{\ell_n}}{2 - 3\frac{\lambda_n}{\ell_n}}$$

$$\lambda_n = \frac{\Sigma\varepsilon - \varepsilon_n}{6E \frac{I_n}{\ell_n} + 3(\Sigma\varepsilon - \varepsilon_n)} \, \ell_n$$

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